Mathematics: As a Key Factor to Master the Challenges of the Energy Transition

Federal Ministry of Education and Research

Mathematics for Innovations as Contribution to Energy Transition

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

- Model Order Reduction and Flexibility Information

TenneT
Taking power further

energy saxony

TECHNISCHE UNIVERSITÄT CHEMNITZ

- Robust Model Analysis and Control
- Mixed-Integer and Semi-Definite Power Flow Optimization

KONSENS

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

enso NETZ

VENIOS
Uniting IT and Energy

Consistent Optimization and Stabilization of Electrical Networked Systems

Control of Residential Energy Systems using Energy Storages & Controllable Loads

Philipp Sauerteig
Technische Universität Ilmenau

joint work with
Philipp Braun (University of Newcaslte), Sara Grundel (MPI Magdeburg),
Karl Worthmann (TU Ilmenau)

funded by
the Federal Ministry of Education and Research

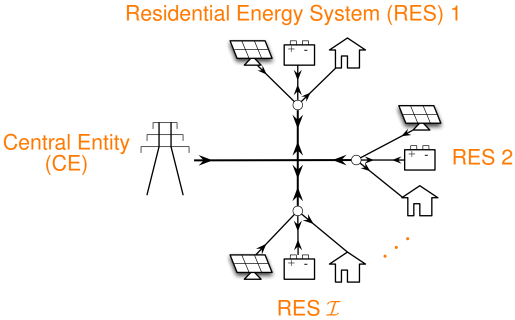
Federal Ministry
of Education
and Research

20th European Conference on Mathematics for Industry
Budapest, 22nd June 2018

Outline

- Modelling Residential Energy Systems
- Excursus: Model Predictive Control
- Distributed Optimization via ADMM
- Numerical Results
- Outlook

Modelling Residential Energy Systems



Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)
System equation of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \dots, \mathcal{I}\}$ at time $k \in \mathbb{N}_0$:

$$\begin{aligned} x_i(k+1) \\ z_i(k) \end{aligned}$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery
- Power demand $z_i(k) \in \mathbb{R}$

Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)
System equation of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \ldots, \mathcal{I}\}$ at time $k \in \mathbb{N}_0$:

$$\begin{aligned} x_i(k+1) &= \quad x_i(k) + T(\quad u_i^+(k) + u_i^-(k)) \\ z_i(k) & \end{aligned}$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery
- Power demand $z_i(k) \in \mathbb{R}$

- Charging rate $u_i^+(k) \geq 0$ and discharging rate $u_i^-(k) \leq 0$
- Sampling interval length $T > 0$

Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)
System equation of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \ldots, \mathcal{I}\}$ at time $k \in \mathbb{N}_0$:

$$\begin{aligned} x_i(k+1) &= x_i(k) + T(u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + u_i^-(k) \end{aligned}$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery
- Power demand $z_i(k) \in \mathbb{R}$
- Net consumption $w_i(k) = \ell_i(k) - g_i(k) \in \mathbb{R}$ (load minus generation)
- Charging rate $u_i^+(k) \geq 0$ and discharging rate $u_i^-(k) \leq 0$
- Sampling interval length $T > 0$

Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)
System equation of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \ldots, \mathcal{I}\}$ at time $k \in \mathbb{N}_0$:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \end{aligned}$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery
- Power demand $z_i(k) \in \mathbb{R}$
- Net consumption $w_i(k) = \ell_i(k) - g_i(k) \in \mathbb{R}$ (load minus generation)
- Charging rate $u_i^+(k) \geq 0$ and discharging rate $u_i^-(k) \leq 0$
- Sampling interval length $T > 0$
- Losses $\alpha_i, \beta_i, \gamma_i \in (0, 1]$ due to energy transformation

Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

$$0 \leq \qquad x_i(k) \qquad \leq C_i$$

Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

$$\begin{aligned} 0 \leq \quad & x_i(k) & \leq C_i \\ \underline{u}_i \leq \quad & u_i^-(k) & \leq 0 \\ 0 \leq \quad & u_i^+(k) & \leq \overline{u}_i \end{aligned}$$

Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

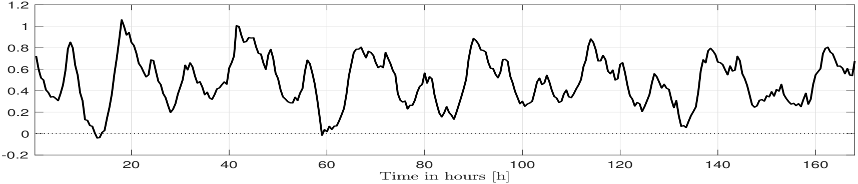
$$
\begin{aligned}
0 \leq & \quad x_i(k) & \leq C_i \\
\underline{u}_i \leq & \quad u_i^-(k) & \leq 0 \\
0 \leq & \quad u_i^+(k) & \leq \overline{u}_i \\
0 \leq & \quad \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} & \leq 1
\end{aligned}
$$

Basic Setting

Constraints: For all $i \in \mathbb{N}_\mathcal{I}$ and all $k \in \mathbb{N}_0$

$$
\begin{aligned}
0 \leq\ & x_i(k) && \leq C_i \\
\underline{u}_i \leq\ & u_i^-(k) && \leq 0 \\
0 \leq\ & u_i^+(k) && \leq \overline{u}_i \\
0 \leq\ & \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} && \leq 1
\end{aligned}
$$

Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} w_i$

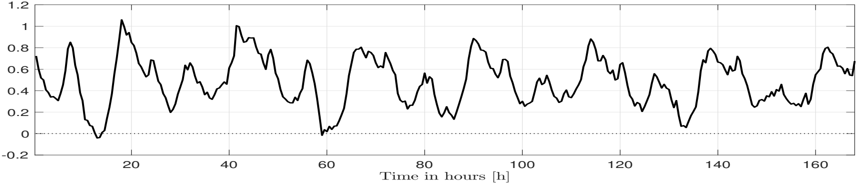


Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

$$\begin{aligned}
0 \leq \quad & x_i(k) & \leq C_i \\
\underline{u}_i \leq \quad & u_i^-(k) & \leq 0 \\
0 \leq \quad & u_i^+(k) & \leq \overline{u}_i \\
0 \leq \quad & \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} & \leq 1
\end{aligned}$$

Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} w_i$



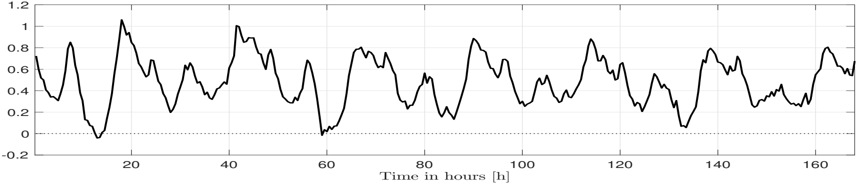
Problem: Fluctuations of the power demand

Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

$$\begin{aligned} 0 \leq \quad & x_i(k) & \leq C_i \\ \underline{u}_i \leq \quad & u_i^-(k) & \leq 0 \\ 0 \leq \quad & u_i^+(k) & \leq \overline{u}_i \\ 0 \leq \quad & \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} & \leq 1 \end{aligned}$$

Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} w_i$



Problem: Fluctuations of the power demand

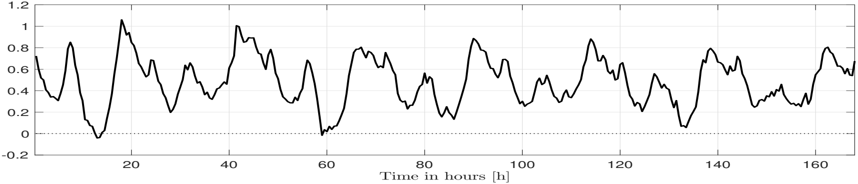
Idea: Exploit flexibilities: storage devices

Basic Setting

Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

$$\begin{aligned} 0 \leq && x_i(k) && \leq C_i \\ \underline{u}_i \leq && u_i^-(k) && \leq 0 \\ 0 \leq && u_i^+(k) && \leq \overline{u}_i \\ 0 \leq && \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} && \leq 1 \end{aligned}$$

Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} w_i$

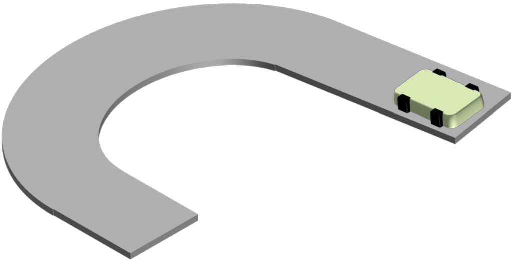


Problem: Fluctuations of the power demand

Idea: Exploit flexibilities: storage devices

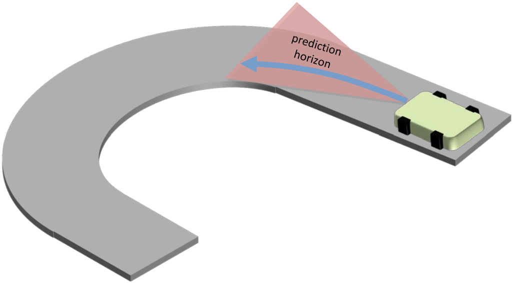
$\rightsquigarrow$ Prediction of the net consumption only on a small time window

Principle of Model Predictive Control



1. Obtain state measurement

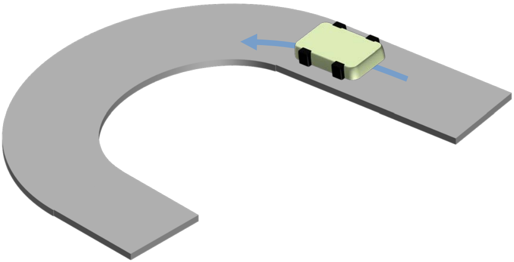
Principle of Model Predictive Control



1. Obtain state measurement

2. Predict system and optimize input

Principle of Model Predictive Control

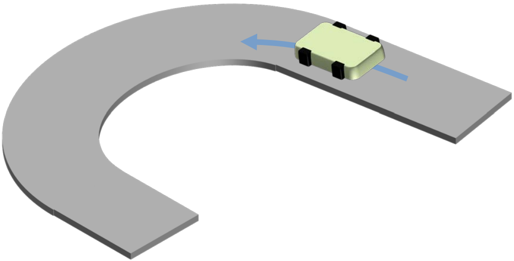


1. Obtain state measurement

2. Predict system and optimize input

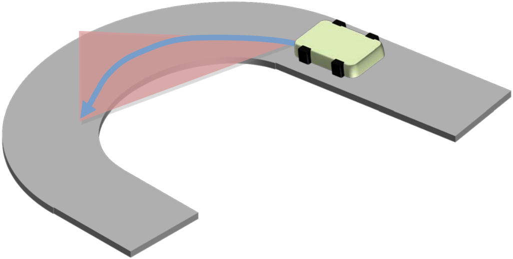
3. Apply “optimal“ input value

Principle of Model Predictive Control



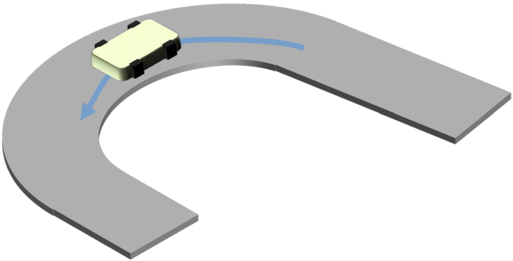
1. Obtain state measurement
2. Predict system and optimize input
3. Apply “optimal“ input value

Principle of Model Predictive Control



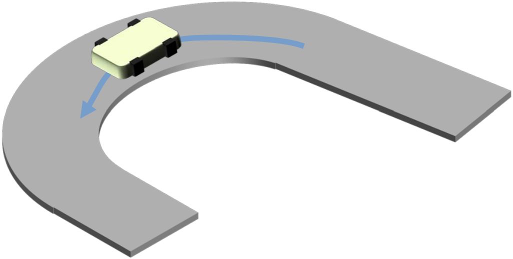
1. Obtain state measurement
2. Predict system and optimize input
3. Apply “optimal“ input value

Principle of Model Predictive Control



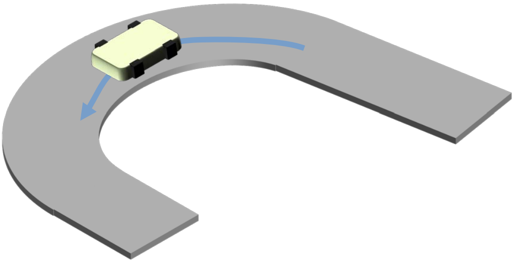
1. Obtain state measurement
2. Predict system and optimize input
3. Apply “optimal“ input value

Principle of Model Predictive Control



1. Obtain state measurement
2. Predict system and optimize input
3. Apply “optimal“ input value

Principle of Model Predictive Control



Control via “repeated” prediction & optimization

Thanks to B. Kern, OVG Universität Magdeburg.

Optimal Control of RESs

Objective: Minimize the deviation from the overall average net consumption

$$\overline{\zeta}(k) = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \zeta_i(k),$$

where

$$\zeta_i(k) = \begin{cases} \frac{1}{k+1} \sum_{n=0}^{k} w_i(n) & \text{if } k < N-1, \\ \frac{1}{N} \sum_{n=k-N+1}^{k} w_i(n) & \text{if } k \geq N-1. \end{cases}$$

Optimal Control of RESs

Objective: Minimize the deviation from the overall average net consumption

$$\bar{\zeta}(k) = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \zeta_i(k),$$

where

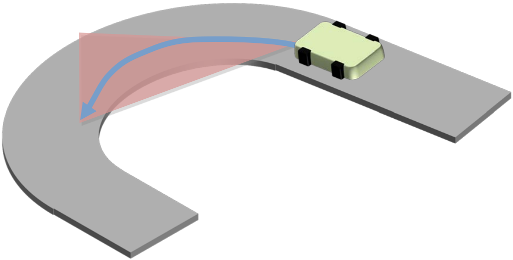
$$\zeta_i(k) = \begin{cases} \frac{1}{k+1} \sum_{n=0}^{k} w_i(n) & \text{if } k < N-1, \\ \frac{1}{N} \sum_{n=k-N+1}^{k} w_i(n) & \text{if } k \geq N-1. \end{cases}$$

Opimization Problem:

$$\min_{\mathbf{u}=(\mathbf{u}^+,\mathbf{u}^-)} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \bar{\zeta}(n) \right)^2$$

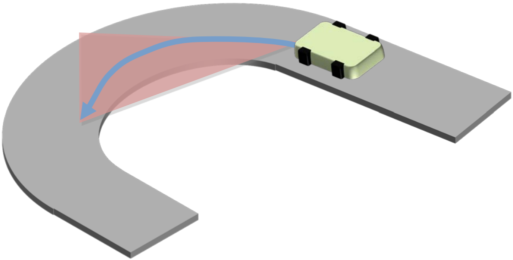
$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Difficulty in MPC



1. Obtain state measurement
2. Predict system and optimize input
3. Apply “optimal“ input value

Difficulty in MPC



Step 2: Solve a large-scale finite-dimensional optimization problem **in every time step**

Distributed computation

$\rightsquigarrow$ Alternating Direction Method of Multipliers (ADMM)

Decoupling of the Objective Function

Optimization Problem:

$$\min_{\mathbf{u}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Decoupling of the Objective Function

Optimization Problem:

$$\min_{\mathbf{u}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Decoupled Formulation:

$$\min_{\mathbf{u},\mathbf{a}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\overline{\mathbf{a}}(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in \{k, \ldots, k+N-1\}$$

- $\mathbf{a}_i = (a_i(k), \ldots, a_i(k+N-1))^\top$, $i \in \mathbb{N}_{\mathcal{I}}$
- $\overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i, \quad \mathbf{a} = (\mathbf{a}_1, \ldots, \mathbf{a}_{\mathcal{I}})^\top$

Decoupling of the Objective Function

Optimization Problem:

$$\min_{\mathbf{u}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$
$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Decoupled Formulation:

$$\min_{\mathbf{u},\mathbf{a}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\overline{\mathbf{a}}(n) - \overline{\zeta}(n)\right)^2 = \frac{1}{N} \left\|\overline{\mathbf{a}} - \overline{\zeta}\right\|_2^2$$
$$\text{s.t.} \quad \text{system dynamics and constraints}$$
$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in \{k, \ldots, k+N-1\}$$

- $\mathbf{a}_i = (a_i(k), \ldots, a_i(k+N-1))^\top,\ i \in \mathbb{N}_{\mathcal{I}}$
- $\overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i, \quad \mathbf{a} = (\mathbf{a}_1, \ldots, \mathbf{a}_{\mathcal{I}})^\top$

Decoupling of the Objective Function

Optimization Problem:

$$\min_{\mathbf{u}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Decoupled Formulation:

$$\min_{\mathbf{u},\mathbf{a}} \quad = \left\| \overline{\mathbf{a}} - \overline{\zeta} \right\|_2^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in \{k, \ldots, k+N-1\}$$

- $\mathbf{a}_i = (a_i(k), \ldots, a_i(k+N-1))^\top,\ i \in \mathbb{N}_{\mathcal{I}}$
- $\overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i, \quad \mathbf{a} = (\mathbf{a}_1, \ldots, \mathbf{a}_{\mathcal{I}})^\top$

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number ℓ_{max} of iterations.
Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)
Loop: While $\ell \leq \ell_{\mathsf{max}}$

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)

Loop: While $\ell \leq \ell_{\max}$

$\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\|\bar{\mathbf{a}} - \bar{\zeta}\right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top(\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2}\left\|\mathbf{z}_i - \mathbf{a}_i\right\|_2^2\right)$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \; \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2}\left\|\mathbf{z}_i - \mathbf{a}_i^\ell\right\|_2^2$$

for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.
Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)
Loop: While $\ell \leq \ell_{\max}$

$\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\| \overline{\mathbf{a}} - \overline{\zeta} \right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2} \|\mathbf{z}_i - \mathbf{a}_i\|_2^2 \right)$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \; \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i - \mathbf{a}_i^\ell \right\|_2^2$$

   for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

2. The CE solves

$$\mathbf{a}^{\ell+1} \in \arg\min_{\mathbf{a}} \; \left\| \overline{\mathbf{a}} - \overline{\zeta} \right\|_2^2 - \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i^{\ell+1} - \mathbf{a}_\mathbf{i} \right\|_2^2 .$$

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)

Loop: While $\ell \leq \ell_{\max}$

$\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\| \bar{\mathbf{a}} - \bar{\zeta} \right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2} \|\mathbf{z}_i - \mathbf{a}_i\|_2^2 \right)$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \; \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i - \mathbf{a}_i^\ell \right\|_2^2$$

   for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

2. The CE solves

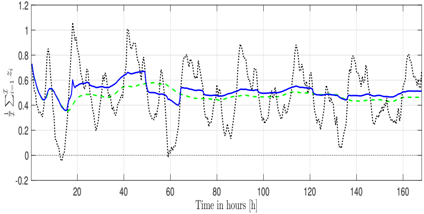
$$\mathbf{a}^{\ell+1} \in \arg\min_{\mathbf{a}} \; \|\bar{\mathbf{a}} - \bar{\zeta}\|_2^2 - \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i^{\ell+1} - \mathbf{a_i} \right\|_2^2 .$$

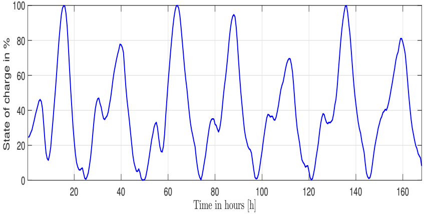
3. The CE updates the Lagrange multipliers

$$\lambda_i^{\ell+1} = \lambda_i^\ell + \rho(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i^{\ell+1}) \quad \forall\, i \in \{1, \ldots, \mathcal{I}\}$$

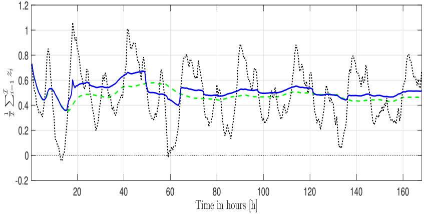
   and broadcasts $(\lambda_i^{\ell+1}, \mathbf{a}_i^{\ell+1})$ to RES $i \in \mathbb{N}_{\mathcal{I}}$. Set $\ell = \ell + 1$.

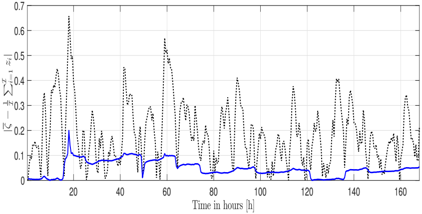
Numerical Simulations

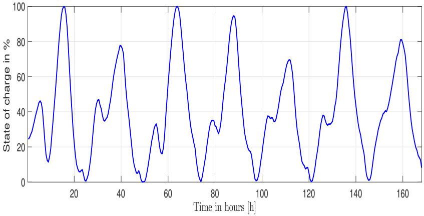




Numerical Simulations







Conclusions:

- Significant peak shaving of the overall performance
- Still room for improvement due to battery capacities & (dis)charging rates

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c. \quad (w_i = \ell_i - g_i)$$

Additional Constraints:

$$0 \leq \quad u_i^c(k) \quad \leq \overline{w}_i^c$$

$$\sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \leq \quad u_i^c(k) \quad \leq \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j)$$

for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

Additional Constraints:

$$\begin{aligned} 0 \leq \quad & u_i^c(k) \quad \leq \overline{w}_i^c \\ \sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \leq \quad & u_i^c(k) \quad \leq \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \end{aligned}$$

for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

System Dynamics:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \end{aligned}$$

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

Additional Constraints:

$$\begin{aligned} 0 \leq \quad & u_i^c(k) \quad \leq \overline{w}_i^c \\ \sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \leq \quad & u_i^c(k) \quad \leq \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \end{aligned}$$

for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

System Dynamics:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i^s(k) + u_i^+(k) + \gamma_i u_i^-(k) + u_i^c(k) \end{aligned}$$

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

Additional Constraints:

$$\begin{aligned} 0 \leq \quad & u_i^c(k) & \leq \overline{w}_i^c \\ \sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \leq \quad & u_i^c(k) & \leq \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \end{aligned}$$

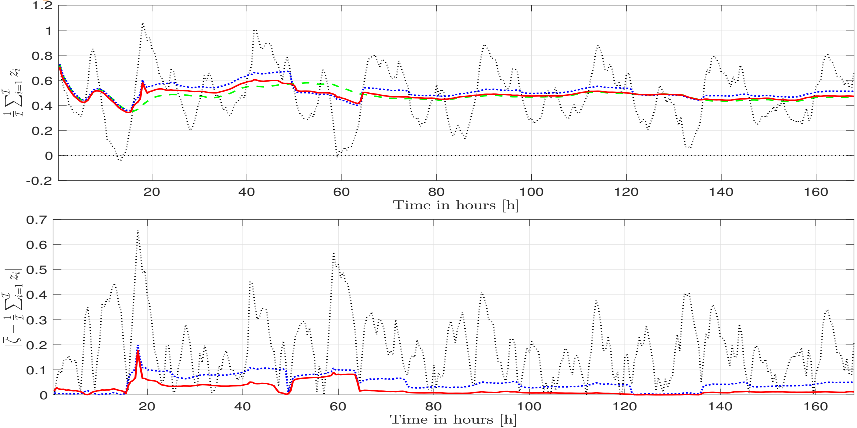
for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

System Dynamics:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i^s(k) + u_i^+(k) + \gamma_i u_i^-(k) + u_i^c(k) \end{aligned}$$

$\rightsquigarrow$ Additional optimization variable u^c

Impact of Controllable Loads

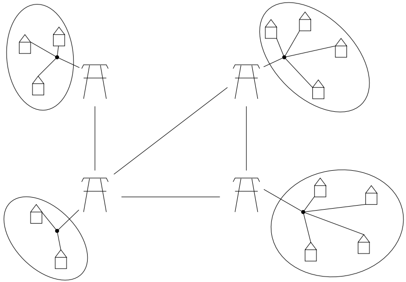


Conclusions:

- Further improvement of the overall performance
- Still no 100% accuracy

Outlook

- Coupled microgrids



First numerical simulations show potential, but no convergence analysis so far.

Outlook

- Coupled microgrids
- Surrogate models
  - For single microgrids

Outlook

- Coupled microgrids
- Surrogate models
  - For single microgrids
- Uncertainties
  - Time series analysis to identify outliners

Outlook

- Coupled microgrids
- Surrogate models
  - For single microgrids
- Uncertainties
  - Time series analysis to identify outliners
- Price-based control
  - Increasing price function $p : \mathbb{R} \to \mathbb{R}$ with $p(0) = 0$
  - First approach: $p(z; c) = Tcz$, $c > 0$ chosen by CE
    $\Rightarrow$ no strong convexity
    $\Rightarrow$ no convergence of dual ascent algorithm guaranteed

Outlook

- Coupled microgrids
- Surrogate models
  - For single microgrids
- Uncertainties
  - Time series analysis to identify outliners
- Price-based control
  - Increasing price function $p : \mathbb{R} \to \mathbb{R}$ with $p(0) = 0$
  - First approach: $p(z; c) = Tcz$, $c > 0$ chosen by CE
    $\Rightarrow$ no strong convexity
    $\Rightarrow$ no convergence of dual ascent algorithm guaranteed
  - Remedy: $p(z; c) = Ta\left(z + b(z - c)^2 - bc^2\right)$ with $a, b > 0$, $c \in \mathbb{R}$ (such that p is increasing)

Thank you for your attention!

- P. Braun, P. Sauerteig, K. Worthmann. Distributed optimization based control on the example of microgrids, *Computational Intelligence and Optimization Methods for Control Engineering*, Preprint, 2018
- P. Braun, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Towards price-based predictive control of a small scale electricity network, *International Journal of Control*, 0(**0**), 1-22, 2017
- P. Braun, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Model Predictive Control of Residential Energy Systems Using Energy Storage & Controllable Loads, *Progress in Industrial Mathematics at ECMI 2014. Mathematics in Industry*, **22**, 617-623, 2016
- K. Worthmann, C.M. Kellett, P. Braun, L. Grüne and S.R. Weller. Distributed and decentralized control of residential energy systems incorporating battery storage, *IEEE Trans. Smart Grid*, 6(**4**), 1914-1923, 2015
- P. Braun, , T. Faulwasser, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Hierarchical Distributed ADMM for Predictive Control with Applications in Power Networks, *IFAC Journal of Systems and Control*, **3**, 10-22, 2018