Distributed Control Enforcing Group Sparsity in Smart Grids

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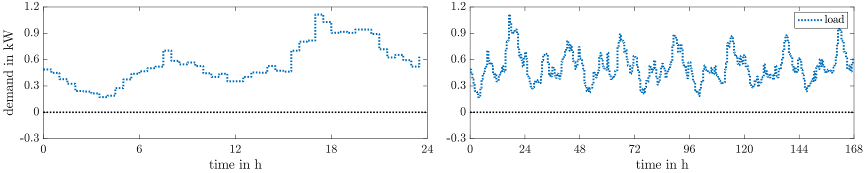
funded by
Federal Ministry of Education and Research

Project: KONSENS (grant: 05M18SIA)

12-17 July 2020, 21st IFAC World Congress, Berlin (virtual)

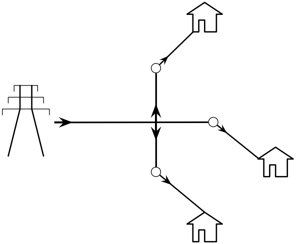
Motivation

Aggregated power demand



(50 households over 1 day/week, data provided by Australian grid operator *Ausgrid*)

Smart grid



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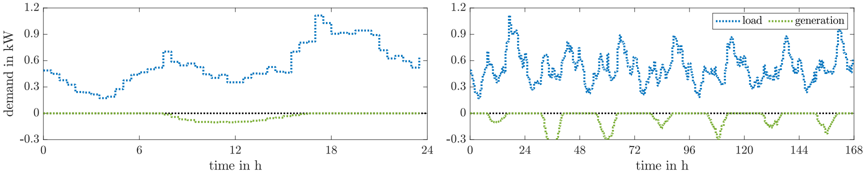

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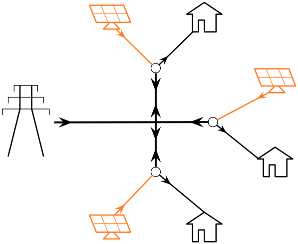
Integration of renewables (PV)

Smart grid



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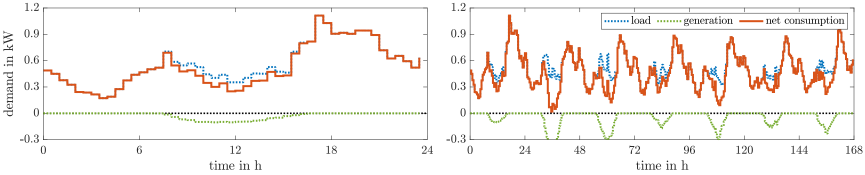

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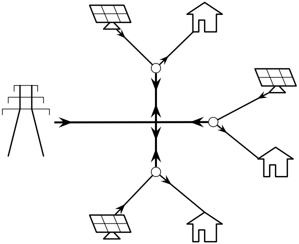
Integration of renewables (PV)

- **problem:** increased volatility of demand

Smart grid



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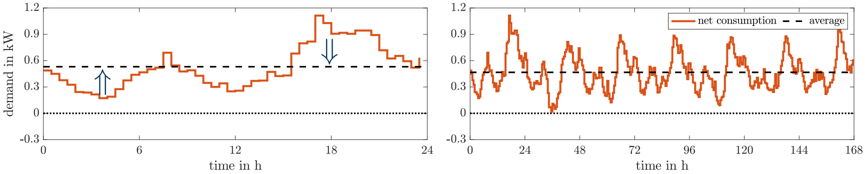

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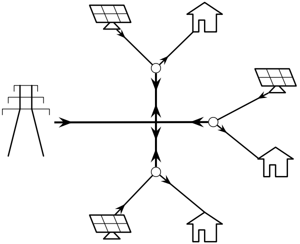
Integration of renewables (PV)

- **problem:** increased volatility of demand
- **goal 1:** constant control energy

Smart grid



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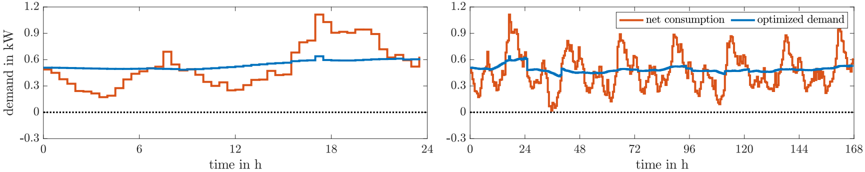

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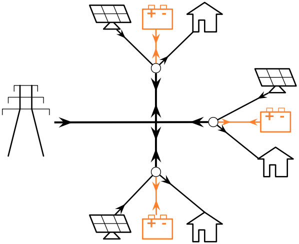
Integration of renewables (PV)

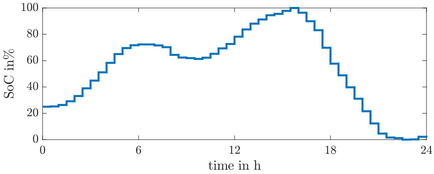
- **problem:** increased volatility of demand
- **goal 1:** constant control energy
- **approach:** exploit energy storage devices

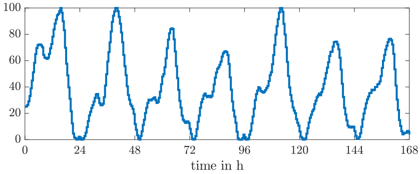
Smart grid



Motivation

Average State of Charge





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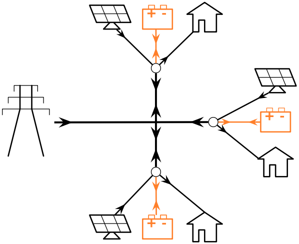
Integration of renewables (PV)

- **problem:** increased volatility of demand
- **goal 1:** constant control energy
- **approach:** exploit energy storage devices
- **goal 2:** reduce charging cycles

Smart grid



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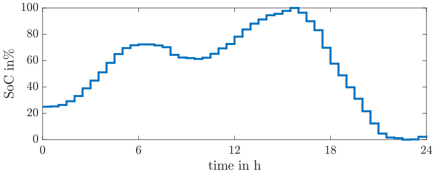

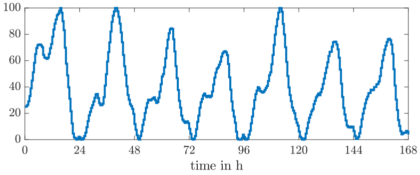

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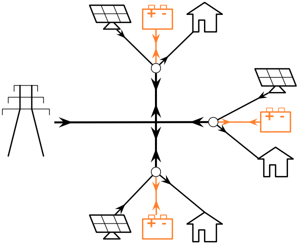
Integration of renewables (PV)

- **problem:** increased volatility of demand
- **goal 1:** constant control energy
- **approach:** exploit energy storage devices
- **goal 2:** reduce charging cycles
- **today's talk:** find a compromise

Smart grid



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Modelling Residential Energy Systems

(see also Worthmann et al. 2014)

Dynamics of *i*-th system, $i \in \{1, \ldots, \mathcal{I}\}$

$$x_i(n+1)$$

Notation

- SoC $x_i \in \mathbb{R}$

Constraints

$$0 \leq \quad x_i(k) \quad \leq C_i$$

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$$x_i(n+1) = x_i(n) + T(u_i^+(n) + u_i^-(n)), \quad x_i(k) = \hat{x}_i$$

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- SoC $x_i \in \mathbb{R}$
- (dis-)charging rate $u_i = (u_i^+, u_i^-)^\top$
- time step $T > 0$

Constraints

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$$z_i(n)$$

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Problem Formulation

Objective 1: Peak shaving, c.f. Worthmann et al. 2014

$$\frac{1}{N}\sum_{n=k}^{k+N-1}\left(\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} z_i(n)-\zeta(n)\right)^2=\frac{1}{N}\left\|\sum_{i=1}^{\mathcal{I}} A_i u_i - b\right\|_2^2 \quad\rightarrow\quad \min$$

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Objective 2: Group sparsity ($p \in \{1, 2\}$), c.f. Yuan and Lin 2006

$$\|u\|_{p,1}^{*} = \sum_{i=1}^{\mathcal{I}} \sigma_i \|u_i\|_p \quad \rightarrow \quad \min$$

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Sparse Optimal Control Problem (OCP)

$$\begin{aligned} \min_{v,u} \quad & \frac{1}{N}\left\|\sum_{i=1}^{\mathcal{I}} A_i v_i - b\right\|_2^2 + \kappa \sum_{i=1}^{\mathcal{I}} \sigma_i \|u_i\|_p \\ \text{s.t.} \quad & v_i = u_i \quad | \; \lambda_i \quad i \in \{1, \ldots, \mathcal{I}\} \\ & D_i u_i \leq d_i \quad i \in \{1, \ldots, \mathcal{I}\} \end{aligned}$$

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ADMM for Sparse OCP (1/2)

(see also Boyd et al. 2011)

Input: initial guesses (u^0, v^0, λ^0), step size $\rho^0 > 0$, stop tolerance $\varepsilon > 0$, tuning parameter $\eta, \mu > 0$.

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For $m = 0 : \mathrm{MaxIte}$

1. *Parallel Step:* Compute for all $i \in \{1, \dots, \mathcal{I}\}$ in parallel

$$u_i^{m+1} = \underset{D_i u_i \leq d_i}{\arg\min} \quad \tilde{\sigma}_i \|u_i\|_p + \frac{\rho^m}{2} \left\| u_i - \frac{\lambda_i^m}{\rho^m} - v_i^m \right\|_2^2$$

$$\lambda_i^{m+1} = \lambda_i^m + \rho^m (v_i^m - u_i^{m+1})$$

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2. *Consensus Step:* Solve unconstrained QP (analytically)

$$v^{m+1} = \underset{v}{\arg\min} \frac{1}{N} \|Av - b\|_2^2 + \frac{\rho}{2} \left\| v - u^{m+1} + \frac{\lambda^{m+1}}{\rho} \right\|_2^2$$

ADMM for Sparse OCP (2/2)

For $m = 0 : \mathrm{MaxIte}$

1. *Parallel Step*
2. *Consensus Step*

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ADMM for Sparse OCP (2/2)

For $m = 0 : \mathrm{MaxIte}$

1. *Parallel Step*
2. *Consensus Step*
3. *Stop Criterion:* If $r^{\mathrm{pri}} \leq \varepsilon$ and $r^{\mathrm{dual}} \leq \varepsilon$, then terminate, where

$$r^{\mathrm{pri}} = \rho \| u^{m+1} - v^{m+1} \|_2 \ , \ r^{\mathrm{dual}} = \rho \| (v^{m+1} - v^m) \|_2 .$$

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4. *Adaptive Dual Step Size:* Update ρ^{m+1} by

$$\rho^{m+1} = \begin{cases} \eta \rho^m & \text{if } r^{\mathrm{pri}} \geq \mu r^{\mathrm{dual}} \\ \rho^m / \eta & \text{if } r^{\mathrm{dual}} \geq \mu r^{\mathrm{primal}} \\ \rho^m & \text{otherwise} \end{cases}$$

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Output: optimal control sequence $u^\star$ and dual $\lambda^\star$

Local solver (1/2)

Note: parallel step of ADMM

$$u_i^{m+1} = \underset{D_i u_i \leq d_i}{\arg\min} \ \tilde{\sigma}_i \|u_i\|_p + \frac{\rho^m}{2} \left\| u_i - \frac{\lambda_i^m}{\rho^m} - v_i^m \right\|_2^2,$$

i.e. a constrained least absolute shrinkage and selection operator (lasso) problem.

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Two solvers depending on $p \in \{1, 2\}$:

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i.e. a constrained least absolute shrinkage and selection operator (lasso) problem.
Two solvers depending on $p \in \{1, 2\}$:

1. $p = 1$: shift ℓ_1 term into the constraints (Boyd and Vandenberghe 2004) by introducing auxiliary variables $s_i \in \mathbb{R}^{2N}$:

$$\begin{aligned} \min_{s_i, u_i} \quad & \tilde{\sigma}_i \mathbb{1}^\top s + \frac{\rho^m}{2} \left\| u_i - \frac{\lambda_i^m}{\rho^m} - v_i^m \right\|_2^2 \\ \text{s.t.} \quad & D_i u_i \leq d_i \;, \; -s \leq u \leq s \;, \end{aligned}$$

$\rightsquigarrow$ existent QP solvers, e.g. `qpOASES` as in Ferreau et al. 2014.

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Local solver (2/2)

$$u_i^{m+1} = \underset{D_i u_i \leq d_i}{\arg\min} \;\; \tilde{\sigma}_i \|u_i\|_p + \frac{\rho^m}{2} \left\| u_i - \frac{\lambda_i^m}{\rho^m} - v_i^m \right\|_2^2$$

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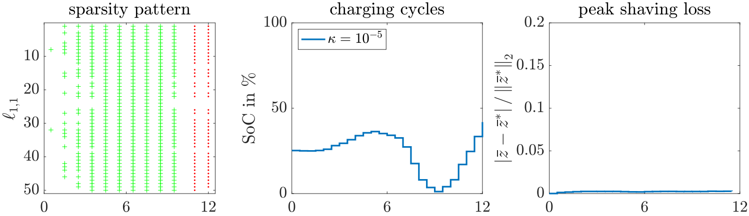
2. $p = 2$: local ADMM solver (Boyd and Vandenberghe 2004):

$$\begin{aligned} s_i &= \mathcal{S}_{\tilde{\sigma}_i/\rho^m} \left(v_i^m + u_i^j + \frac{\lambda_i^m - \xi^j}{\rho^m} \right), \\ u_i^{j+1} &= \underset{D_i u_i \leq d_i}{\arg\min} \; \frac{\rho^m}{2} \left\| u_i - s_i - \frac{\xi_i^j}{\rho^m} \right\|_2^2, \\ \xi_i^{j+1} &= \xi_i^j + \rho^m (u_i^{j+1} - s_i) \end{aligned}$$

with *soft thresholding operator* $\mathcal{S}_a : \mathbb{R}^{2N} \to \mathbb{R}^{2N}$ defined by

$$\mathcal{S}_a(x) = \max\left\{1 - a/\left\|x\right\|_2, 0\right\} x$$

Open-Loop Results



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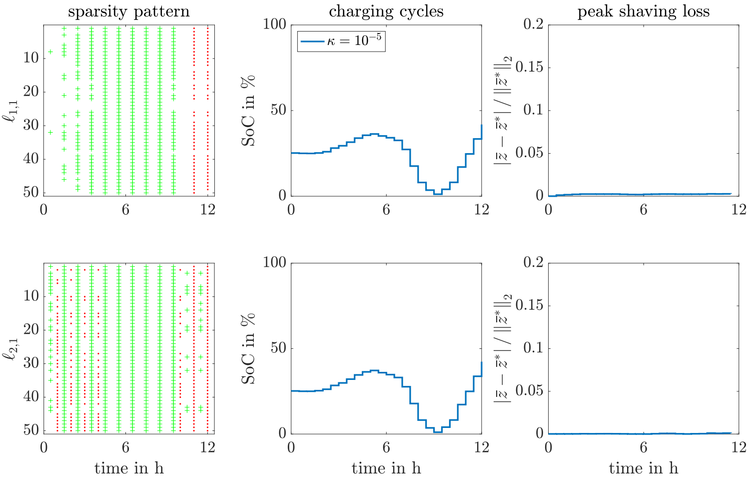

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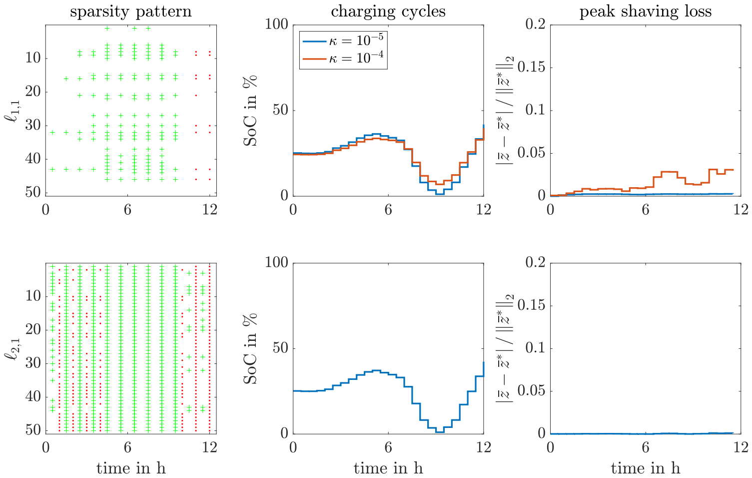

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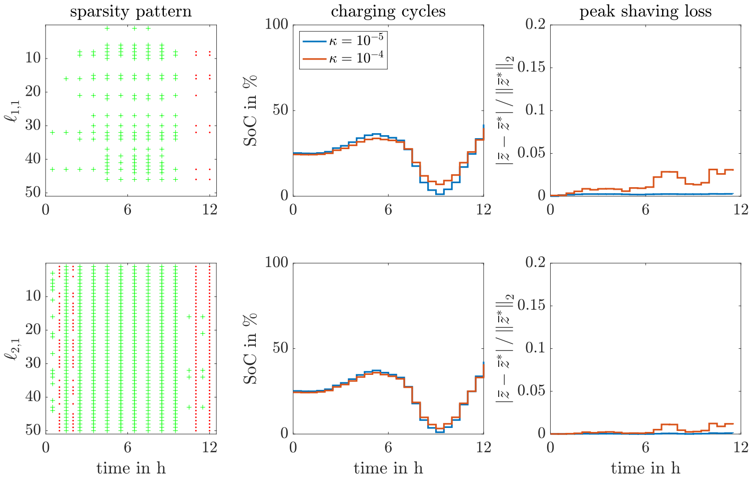
Open-Loop Results



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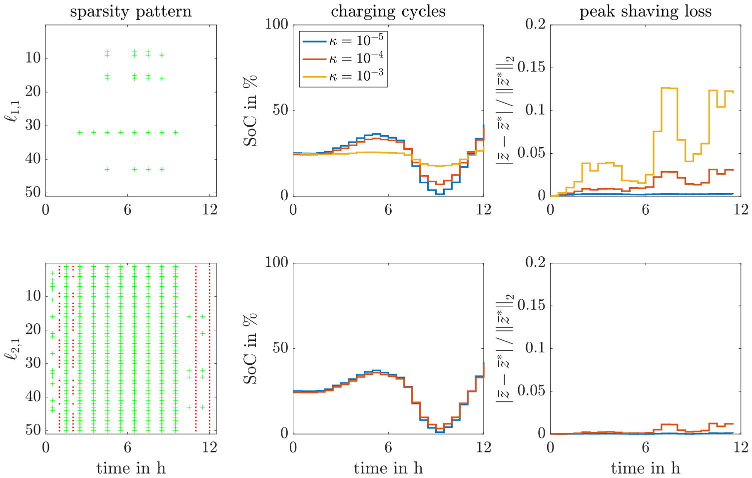

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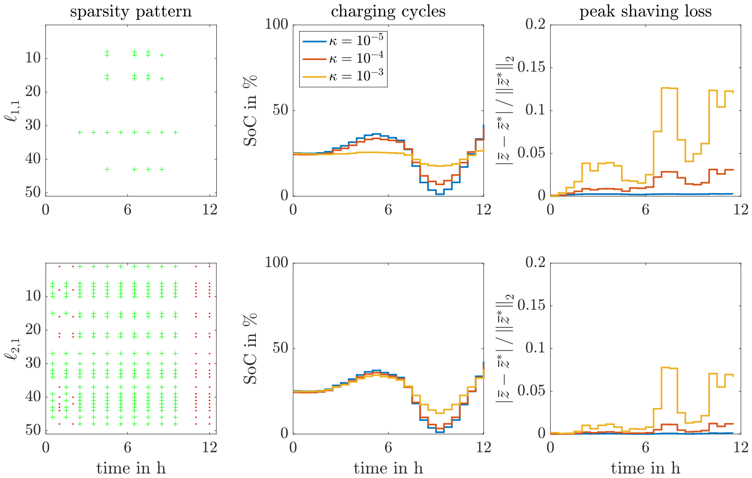

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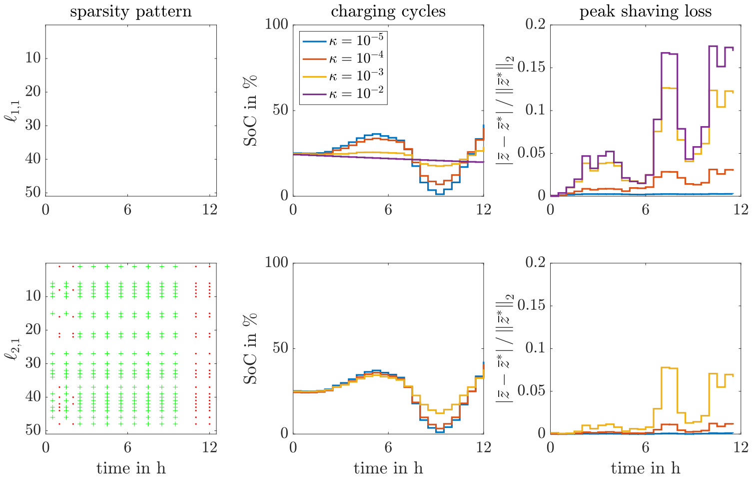

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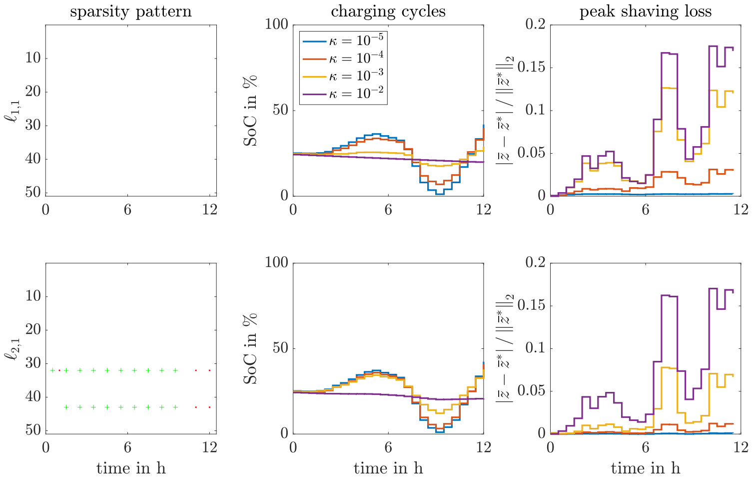

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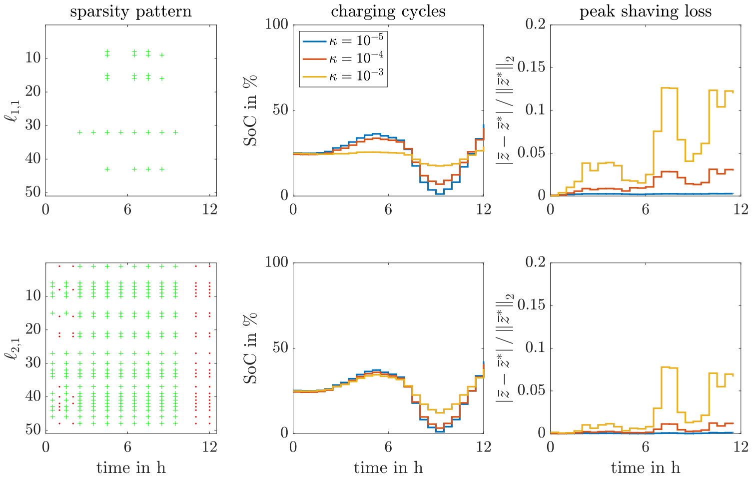

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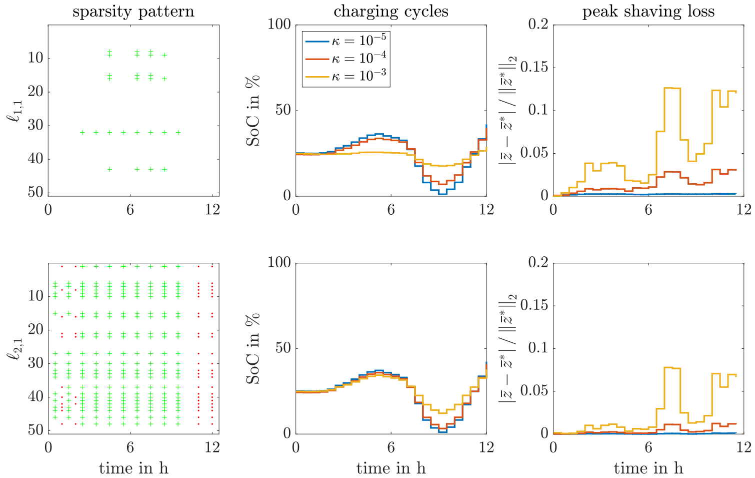

⇝ same batteries are used due to choice of σ_i

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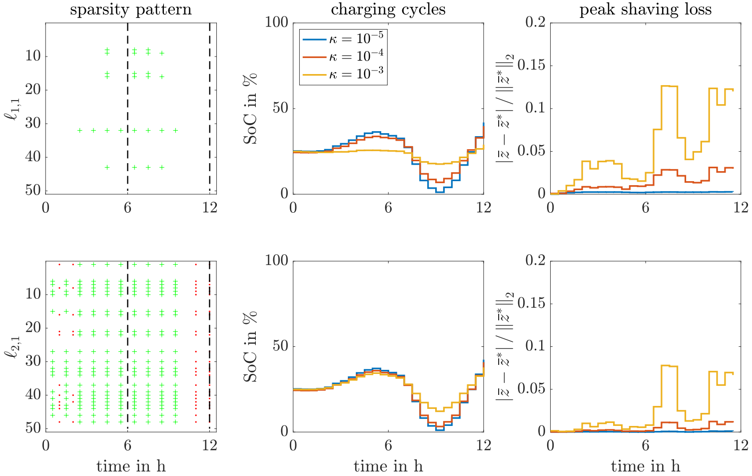

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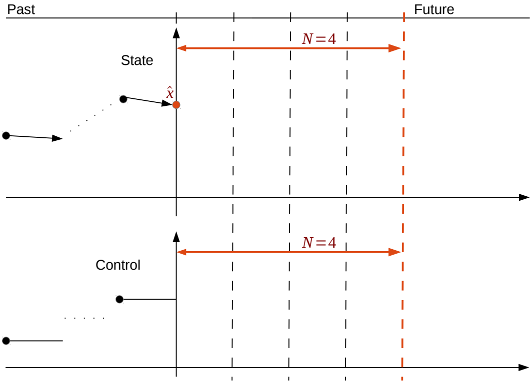
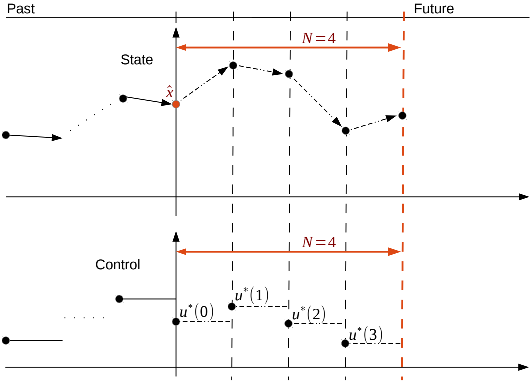
Model Predictive Control



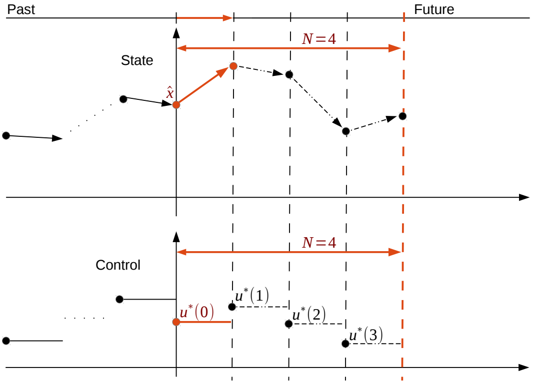
1. Measure current state and predict exogenous quantities.

Model Predictive Control



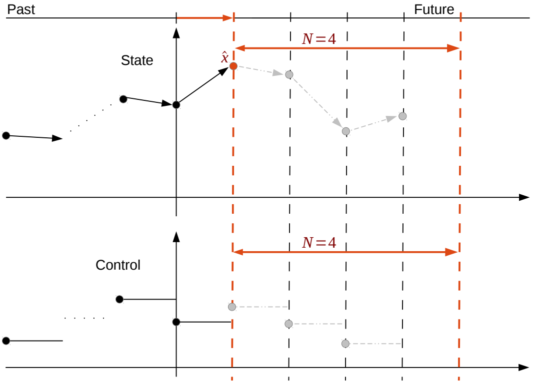
1. Measure current state and predict exogenous quantities.
2. Solve optimal control problem.

Model Predictive Control



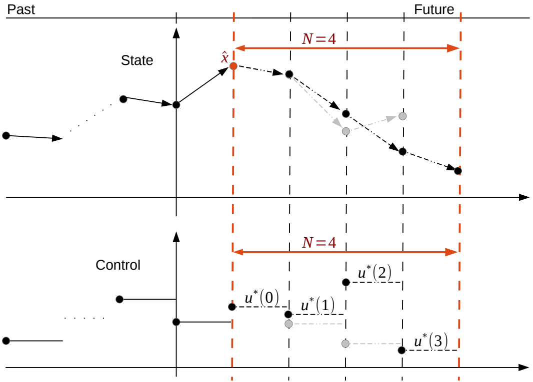
1. Measure current state and predict exogenous quantities.
2. Solve optimal control problem.
3. Implement first control instance.

Model Predictive Control



1. Shift time step, measure current state, and predict exogenous quantities.
2. Solve optimal control problem.
3. Implement first control instance.

Model Predictive Control



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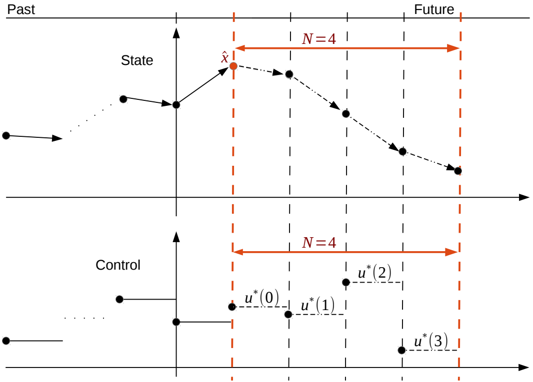

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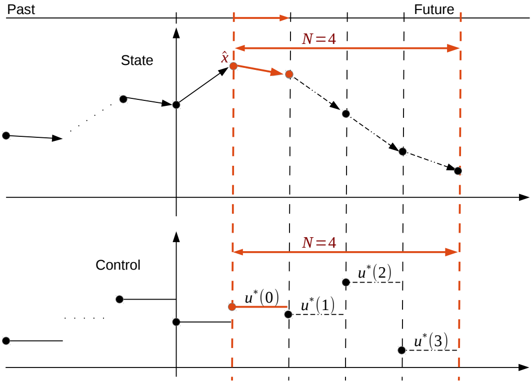

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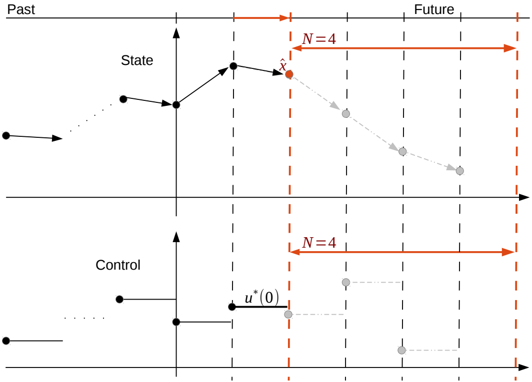

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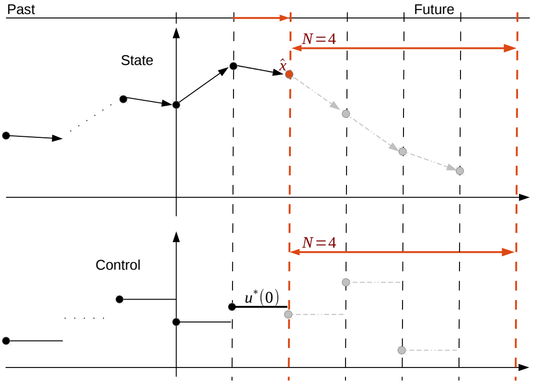

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Closed-Loop Results

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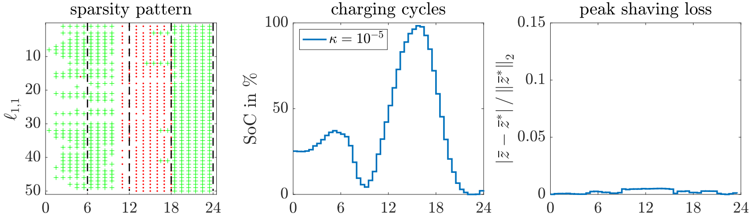

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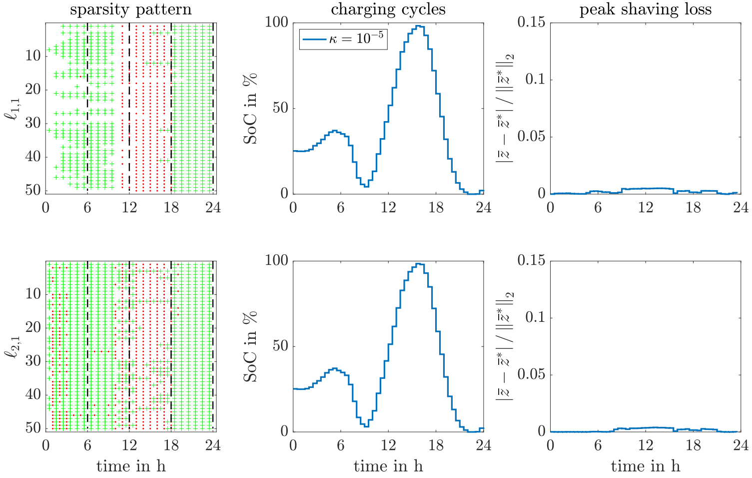

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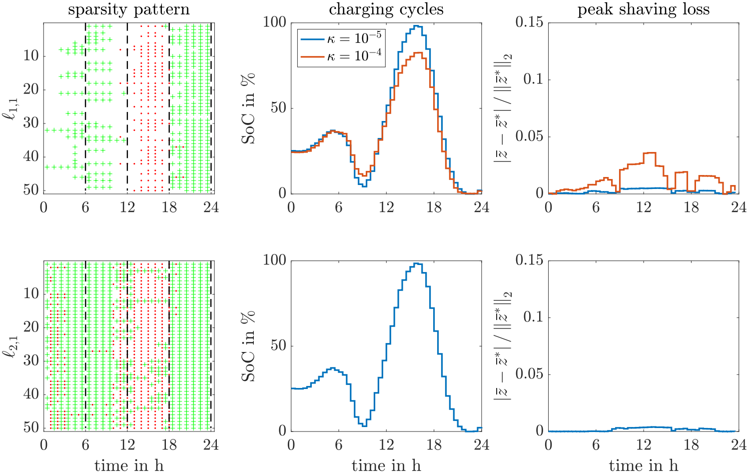

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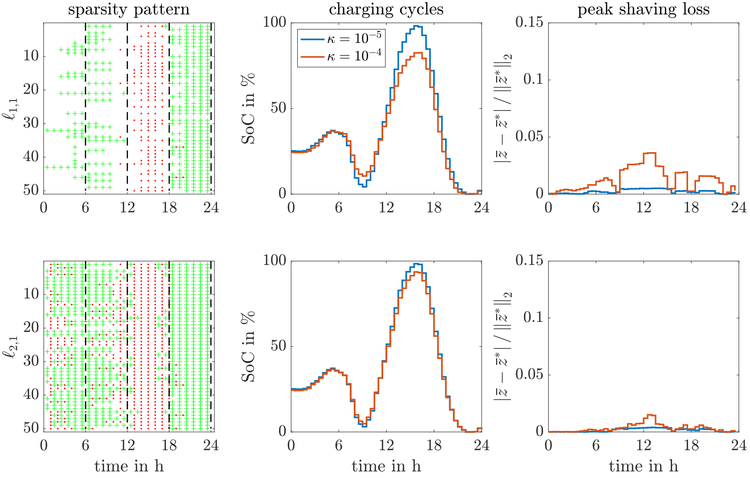

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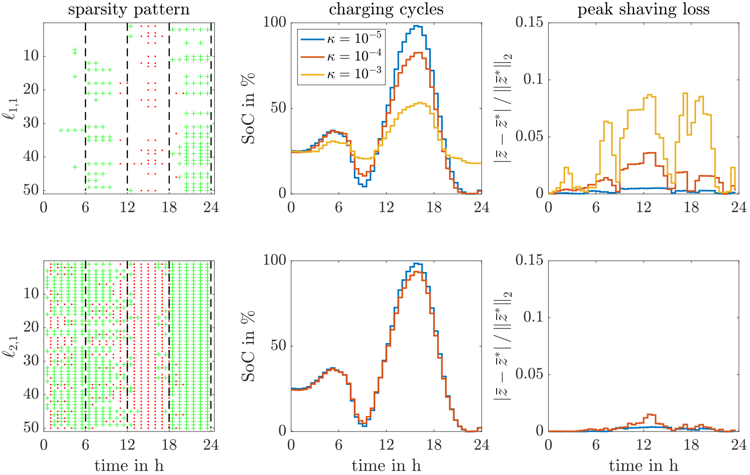

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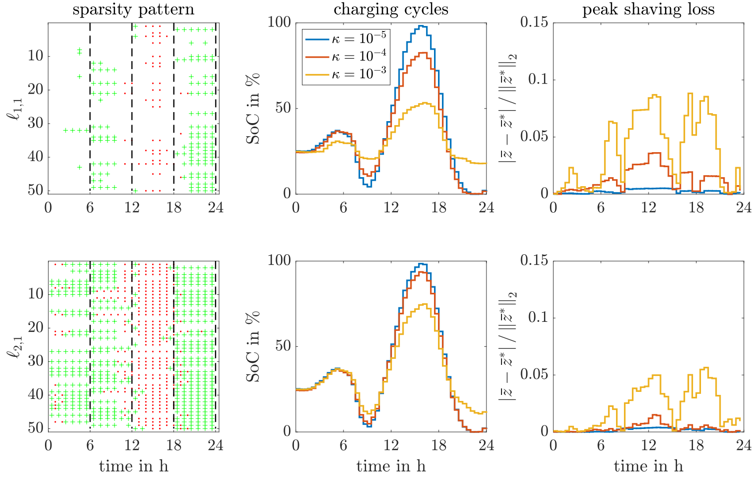

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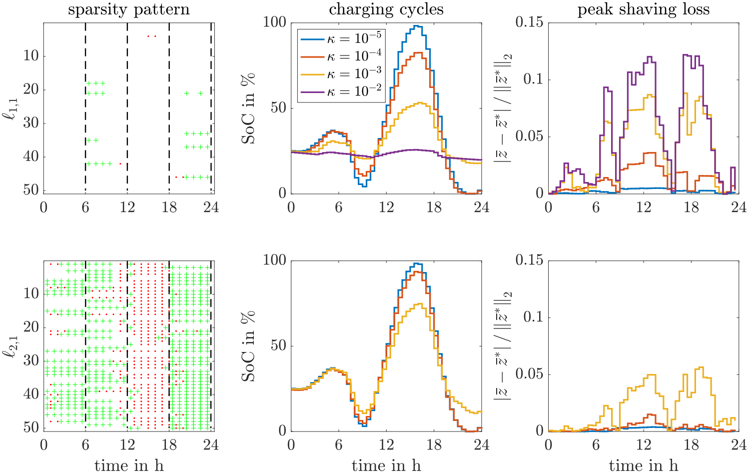

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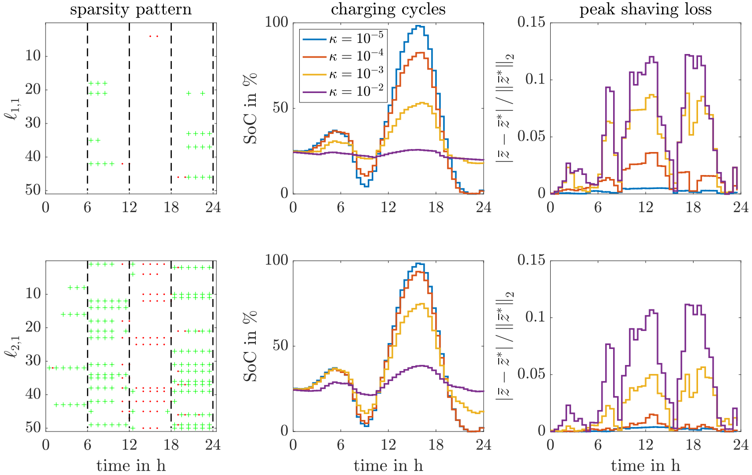

Conclusions & Outlook

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Conclusions & Outlook

What have we achieved?

- a sparse optimal control problem formulation to extend lifetime of batteries
- design a distributed optimization scheme

What might come next?

- a more sophisticated method to choose local weights σ_i
- take net topology into account

References

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Appendix

Parameters for implementation

	expected value	standard deviation
C_i	2.0563 [kWh]	0.2431 [kWh]
$\overline{u}_i$	0.5229 [kW]	0.1563 [kW]
$\underline{u}_i$	−0.5105 [kW]	0.1474 [kW]
α_i	0.9913	0.0053
β_i	0.9494	0.0098
γ_i	0.9487	0.0100

Appendix

Impact of the number of subsystems $\mathcal{I}$ on the percentage of non-zero components of the optimal control $u \in \mathbb{R}^{2N\mathcal{I}}$ for $\kappa = 10^{-3}$

	$\mathcal{I}$	mean	std dev	median
$\ell_{2,1}$	25	30.67	0.85	30.33
	50	24.41	0.46	24.17
	100	18.73	0.26	18.75
$\ell_{1,1}$	25	8.48	0.36	8.38
	50	4.65	0.10	4.65
	100	3.82	0.06	3.81

Distributed Predictive Control Scheme

Offline:

- Initial guess (u^0, λ^0), set $k = 0$, $v^0 = u^0$ and choose weights σ_i and a tolerance $\varepsilon > 0$.

Online:

1) **Subsystems** measure current SoC $x_i(k)$, predict future net consumption w_i and send it to grid operator.
2) **Grid Operator** computes the reference trajectory ζ.
3) Optionally update weights σ_i. Solve sparse OCP to obtain u^* and λ^*.
4) **Subsystems** apply $u_i^*(k)$ if $\|u_i(k)\|_2 \geq \varepsilon$ and 0 otherwise.
5) Reinitialize

$$\begin{aligned} u_i^0 &= \left(u_i^*(k+1)^\top \ \ldots \ u_i^*(k+N-1)^\top \ 0_2^\top\right)^\top \\ \lambda_i^0 &= \left(\lambda_i^*(k+1) \ \ldots \ \lambda_i^*(k+N-1) \ 0\right)^\top \end{aligned}$$

for all $i \in \{1, \ldots, \mathcal{I}\}$. Then, set $k \leftarrow k+1$ and go to Step 1).