Current Research Topics in Distributed Optimization of Smart Grids

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Technische Universität Ilmenau

joint work with
Karl Worthmann (TU Ilmenau)

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of Education
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ShanghaiTech University, 10 June 2019

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	Shanghai
size [km^2]	6 340
population	26 000 000
students	35 000

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size [km^2]	6 340	200
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	Shanghai	Ilmenau	Thuringia
size [km^2]	6 340	200	16 000
population	26 000 000	26 000	2 150 000
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KONSENS: Konsistente Optimierung uNd Stabilisierung Elektrischer NetzwerkSysteme

Federal Ministry of Education and Research

Mathematics for Innovations as Contribution to Energy Transition

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Mathematics for Innovations as Contribution to Energy Transition

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

- Model Order Reduction and Flexibility Information

TECHNISCHE UNIVERSITÄT CHEMNITZ

- Robust Model Analysis and Control
- Mixed-Integer and Semi-Definite Power Flow Optimization

KON SENS

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

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TenneT Taking power further

KON SENS

enso NETZ

energy saxony

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

VENIOS Uniting IT and Energy

Outline

- Optimal Control of Distributed Energy Storage Devices
  - Motivation
  - Modelling Residential Energy Systems
  - Distributed Optimization via ADMM

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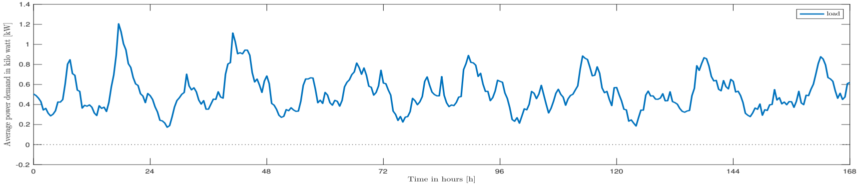
- Optimal Control of Distributed Energy Storage Devices
  - Motivation
  - Modelling Residential Energy Systems
  - Distributed Optimization via ADMM
- Current Research
  - Distributed Optimization via ALADIN
  - Multiobjective Optimization
  - Coupled Microgrids
  - Surrogates

Motivation

Aggregated power demand profile (1 week)



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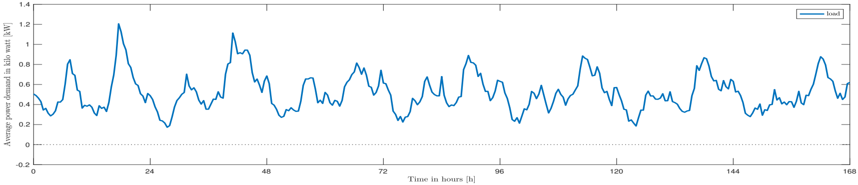
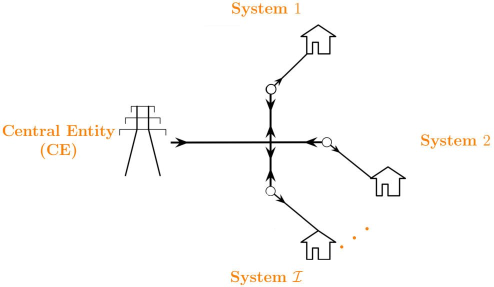


Problem

- volatile load

Smart grid



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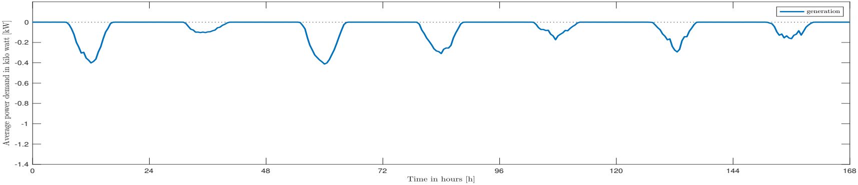

Problem

- volatile load
- generation via photovoltaic

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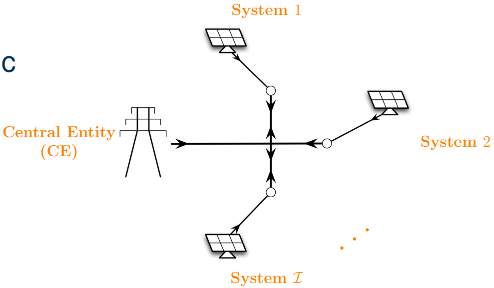

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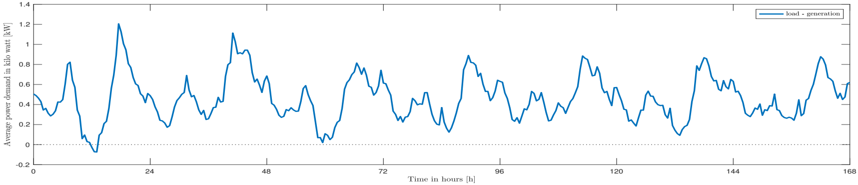

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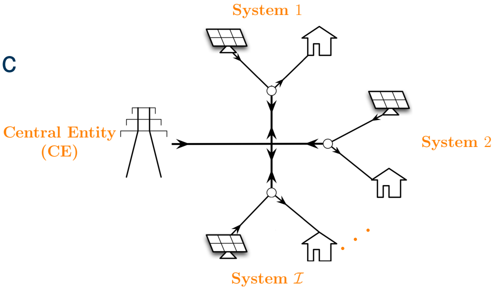

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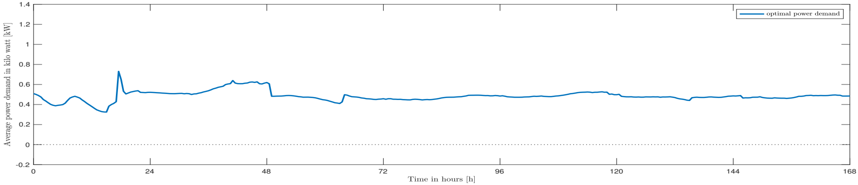

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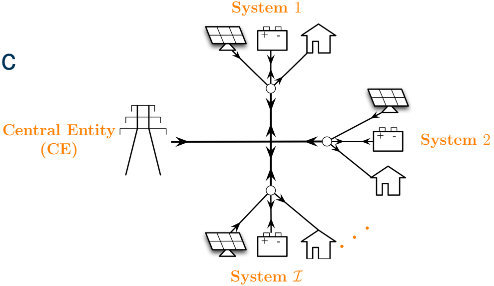
Remedy: exploit flexibilities

- storage devices

Smart grid



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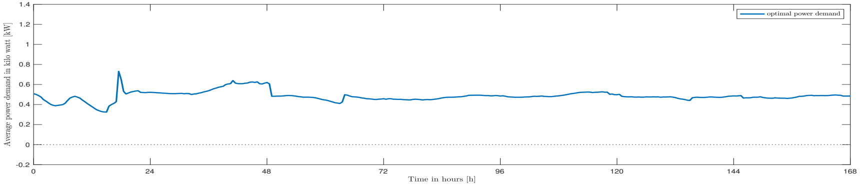

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Remedy: exploit flexibilities

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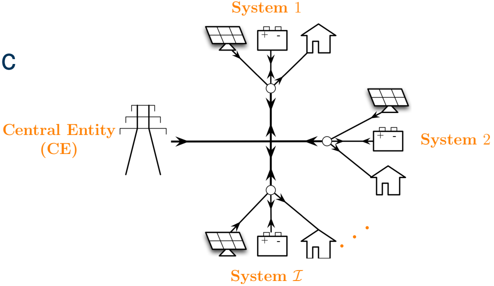
- energy exchange

Smart grid



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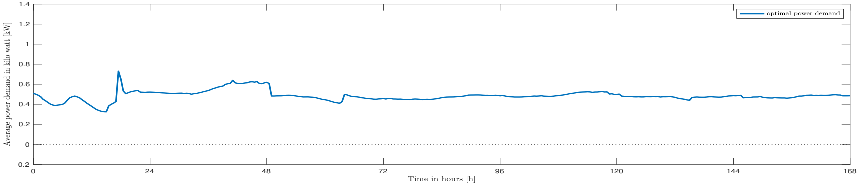

Problem

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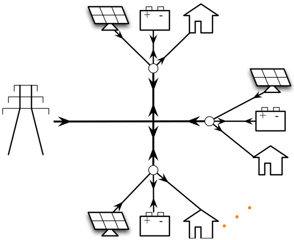
Remedy: exploit flexibilities

- storage devices
- energy exchange
- controllable loads, . . .

Smart grid



Dynamics & Constraints

Given: $\mathcal{I} \in \mathbb{N}$ subsystems (smart homes)

System equation of subsystem $i \in [1 : \mathcal{I}] := \{1, 2, \ldots, \mathcal{I}\}$ at time instants $n \in [k : k + N - 1]$, $k \in \mathbb{N}_0$, $x_i(k) = \hat{x}_i$:

$$x_i(n+1)$$

$$z_i(n)$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$

Constraints: For all

$i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

$$0 \leq \qquad x_i(n) \qquad \leq C_i$$

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$$
\begin{aligned}
x_i(n+1) &= x_i(n) + T(\quad u_i^+(n) + u_i^-(n)) \\
z_i(n) &
\end{aligned}
$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$

Constraints: For all $i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

$$
\begin{aligned}
0 \leq \quad & x_i(n) & \leq C_i \\
\underline{u}_i \leq \quad & u_i^-(n) & \leq 0 \\
0 \leq \quad & u_i^+(n) & \leq \bar{u}_i \\
0 \leq \quad & \frac{u_i^-(n)}{\underline{u}_i} + \frac{u_i^+(n)}{\bar{u}_i} & \leq 1
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$$
\begin{aligned}
x_i(n+1) &= x_i(n) + T(u_i^+(n) + u_i^-(n)) \\
z_i(n) &= w_i(n) + u_i^+(n) + u_i^-(n)
\end{aligned}
$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
- Net consumption $w_i(n) = \ell_i(n) - g_i(n) \in \mathbb{R}$
- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$

Constraints: For all $i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

$$
\begin{aligned}
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$$\begin{aligned} x_i(n+1) &= \alpha_i x_i(n) + T(\beta_i u_i^+(n) + u_i^-(n)) \\ z_i(n) &= w_i(n) + u_i^+(n) + \gamma_i u_i^-(n) \end{aligned}$$

Notation

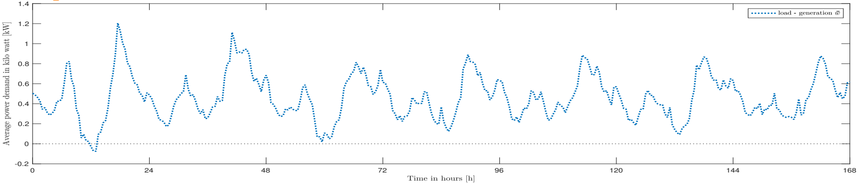
- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
- Net consumption $w_i(n) = \ell_i(n) - g_i(n) \in \mathbb{R}$
- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$
- Efficiencies $\alpha_i, \beta_i, \gamma_i \in (0, 1]$

Constraints: For all

$i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

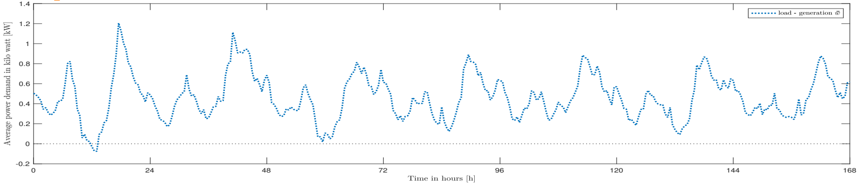
$$\begin{array}{rcl} 0 \leq & x_i(n) & \leq C_i \\ \underline{u}_i \leq & u_i^-(n) & \leq 0 \\ 0 \leq & u_i^+(n) & \leq \bar{u}_i \\ 0 \leq & \dfrac{u_i^-(n)}{\underline{u}_i} + \dfrac{u_i^+(n)}{\bar{u}_i} & \leq 1 \end{array}$$

Objective



Goal: Flatten aggregated power demand profile

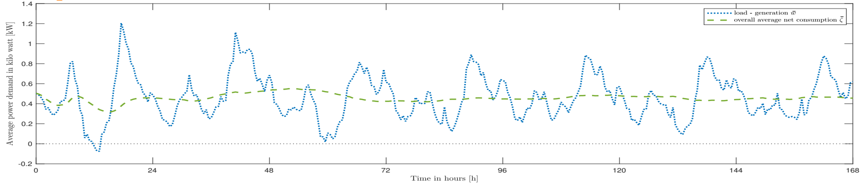
Objective



Goal: Flatten aggregated power demand profile

Idea: Trace some steady reference trajectory

Objective



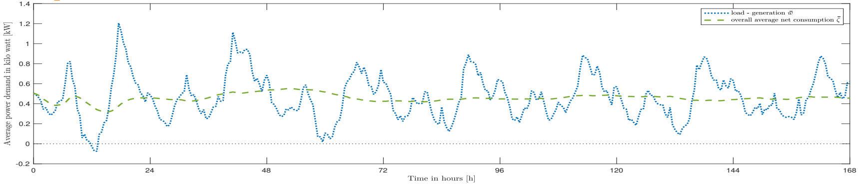
Goal: Flatten aggregated power demand profile

Idea: Trace some steady reference trajectory, e.g. overall average net consumption

$$\bar{\zeta}(n) = \frac{1}{\mathcal{I} \cdot \min\{N_1, n+1\}} \sum_{j=n-\min\{n, N_1-1\}}^{n} \sum_{i=1}^{\mathcal{I}} w_i(j)$$

for some $N_1 \in \mathbb{N}_{\geq 2}$

Objective



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for some $N_1 \in \mathbb{N}_{\geq 2}$

Problem: Future net consumption w_i unknown

$\rightsquigarrow$ prediction of $w_i(j)$, $j \in [k : k + N_2 - 1]$
($N_1 = N_2 = N$ prediction horizon)

Model Predictive Control

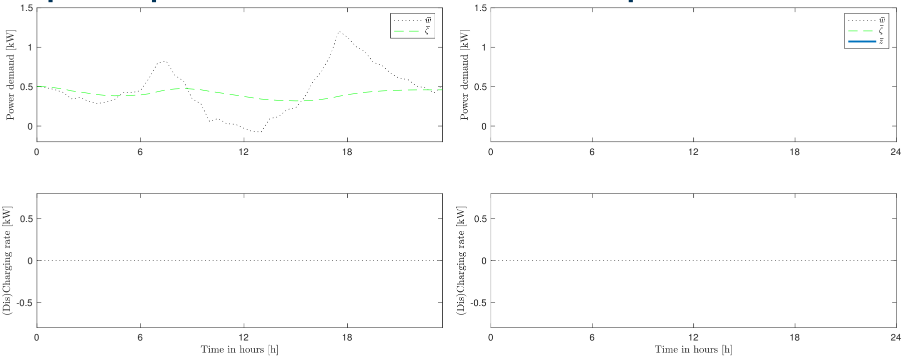
Open loop

Closed loop

Model Predictive Control

Open loop

Closed loop

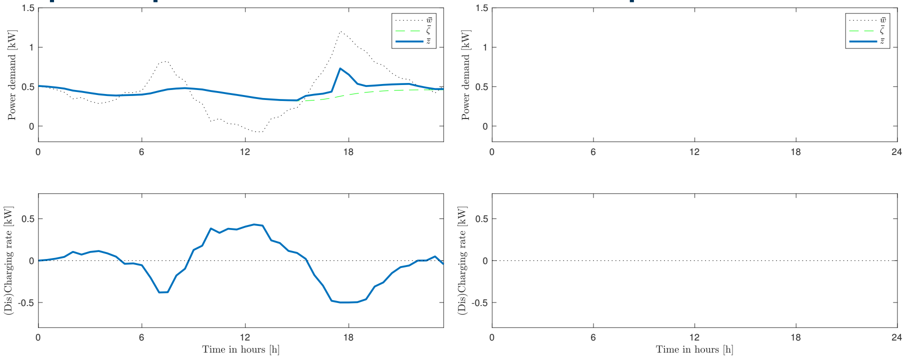


1. Measure current SoC $\hat{x} = x(k)$ and predict (exogenous) input $w = (w(k), \dots, w(k+N-1))^\top$.

Model Predictive Control

Open loop

Closed loop



2. Compute optimal input $u^* = (u^*(k), \ldots, u^*(k+N-1))^\top$.

Model Predictive Control

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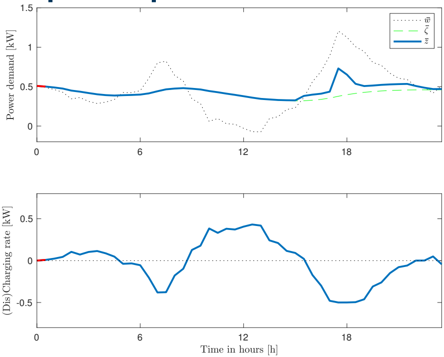
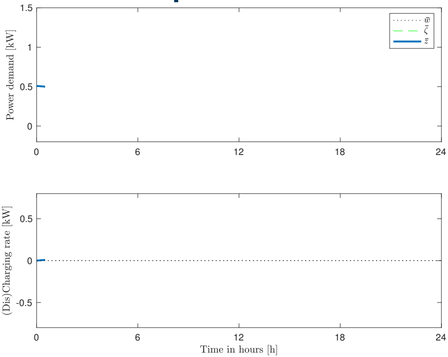


Closed loop



3. Implement $u^*(k)$ and increment k.

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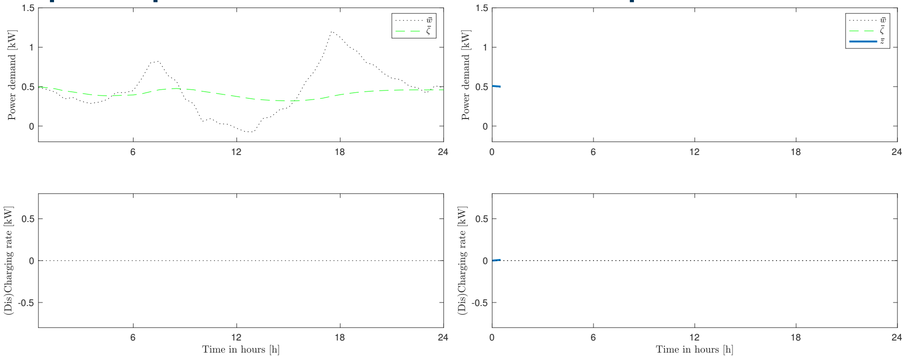

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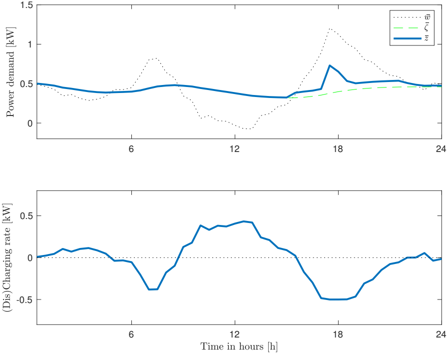

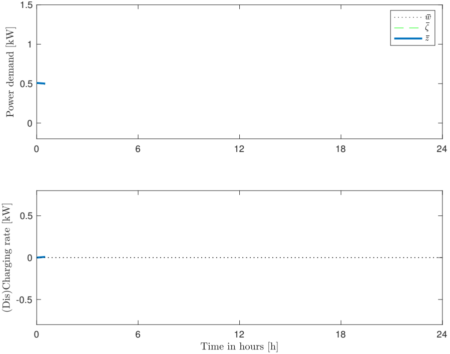

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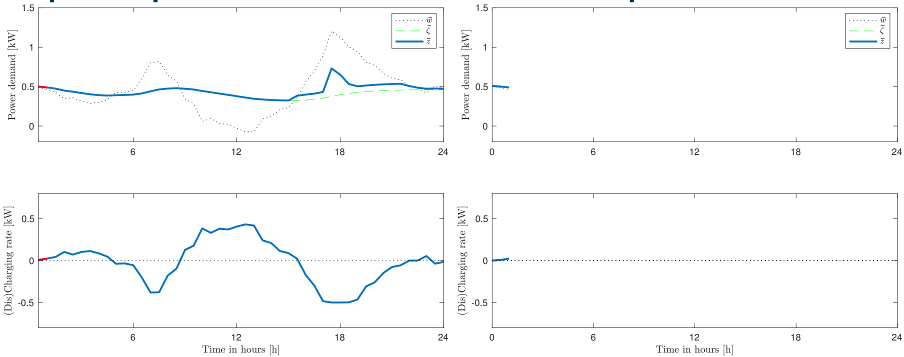

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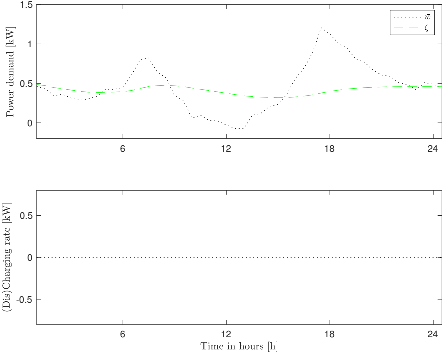

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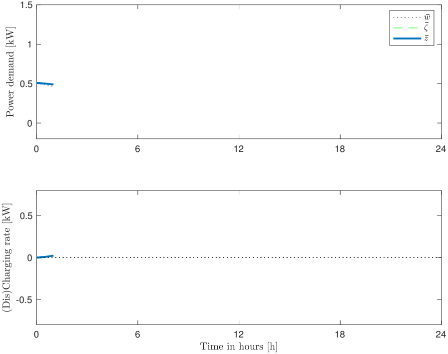

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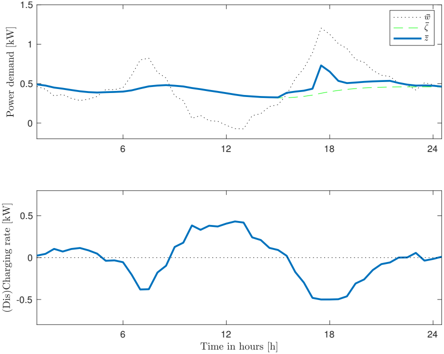

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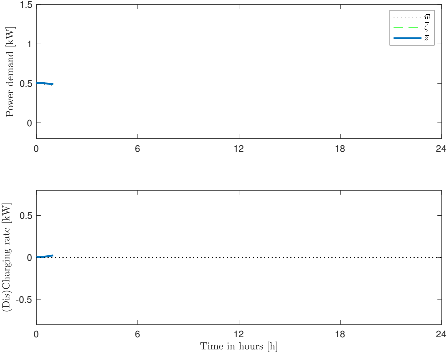

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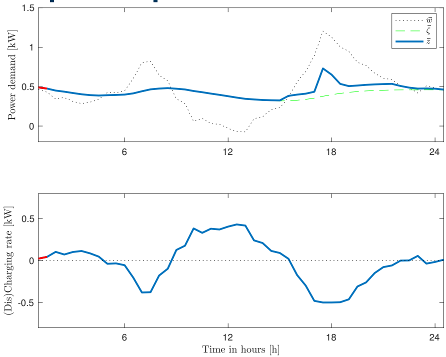

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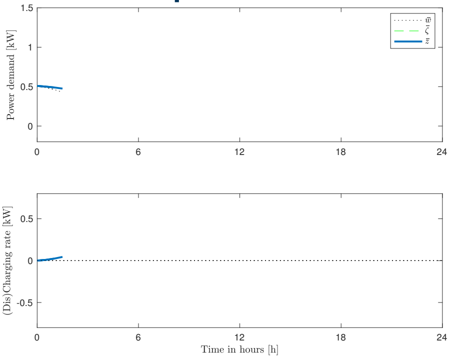

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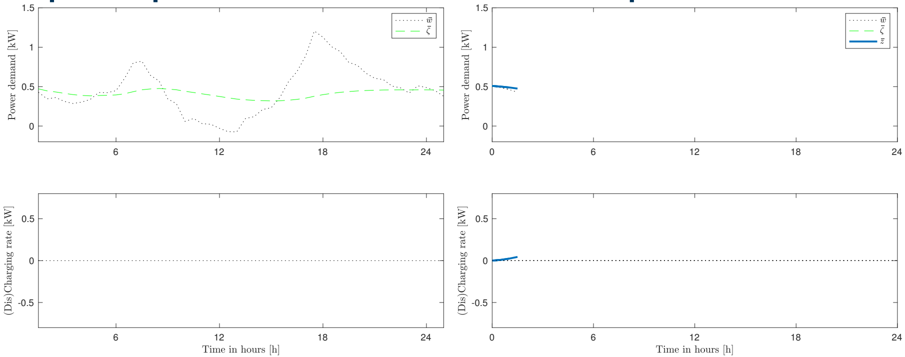

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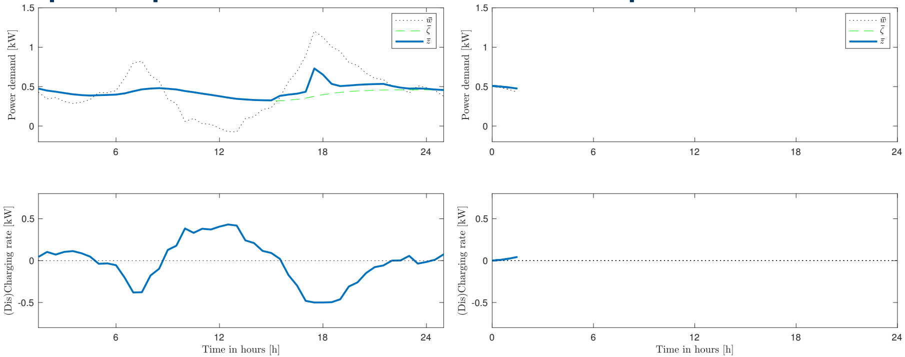

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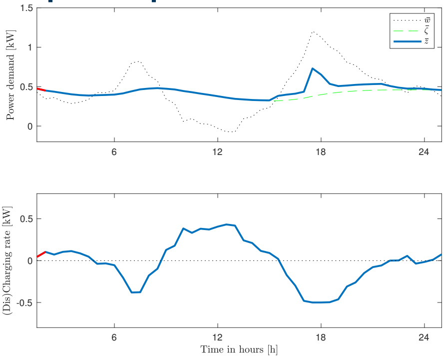

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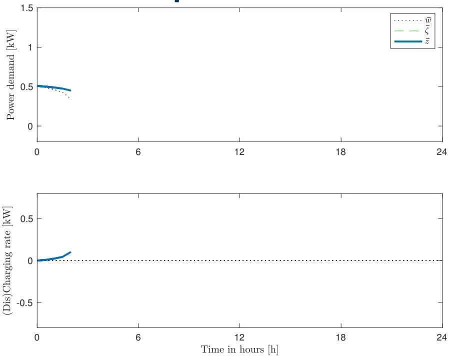

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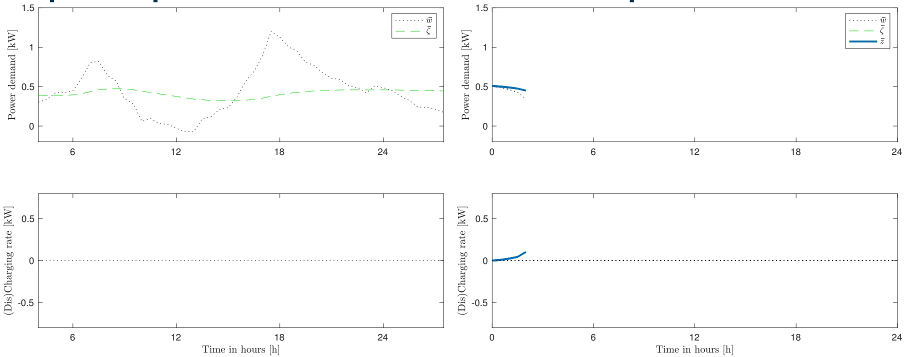

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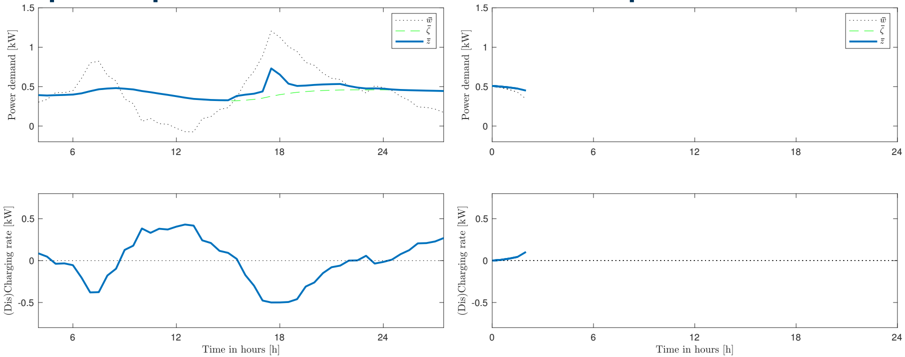

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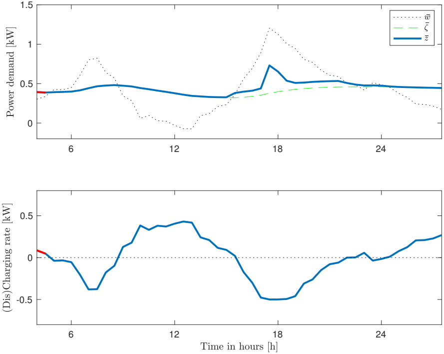
Model Predictive Control

Open loop



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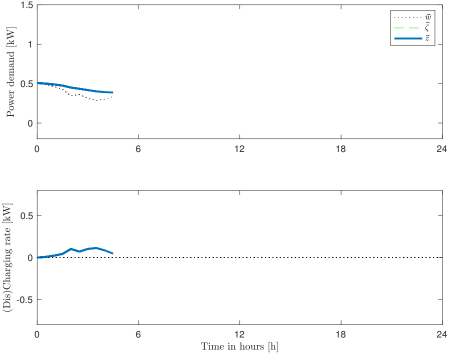

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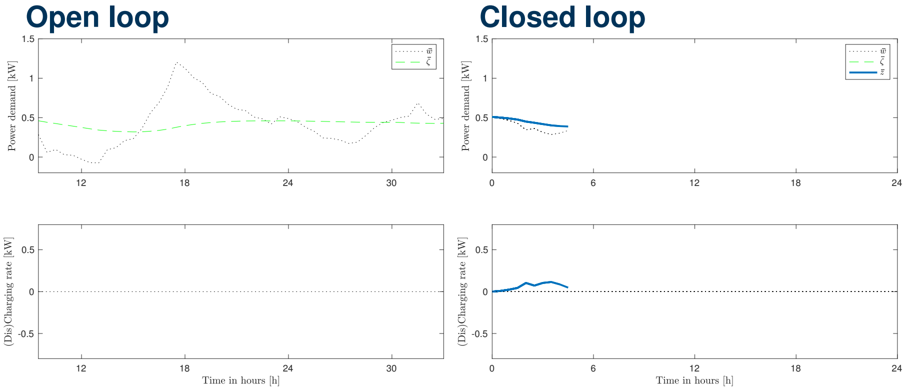

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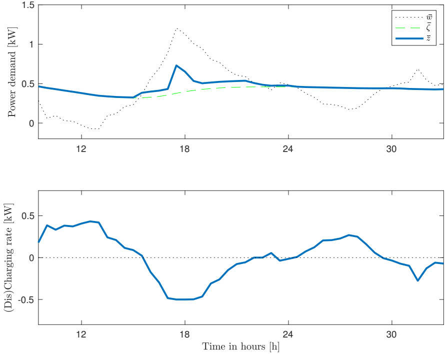

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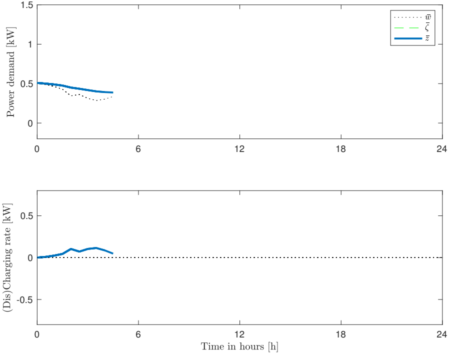

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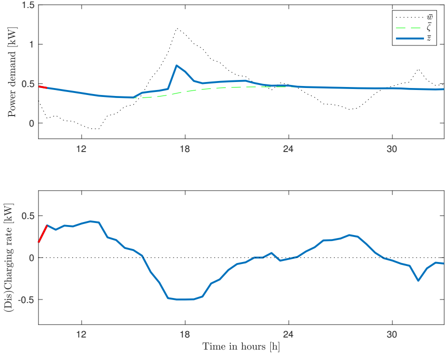

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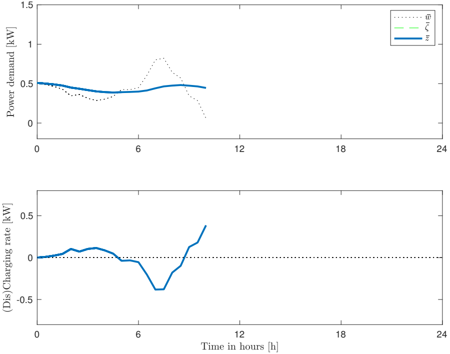

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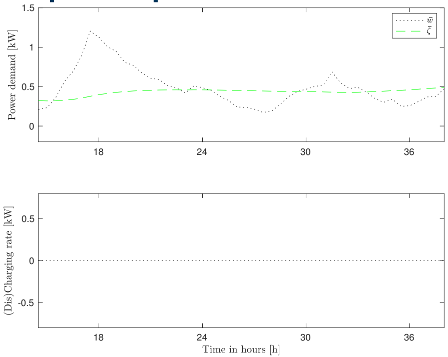

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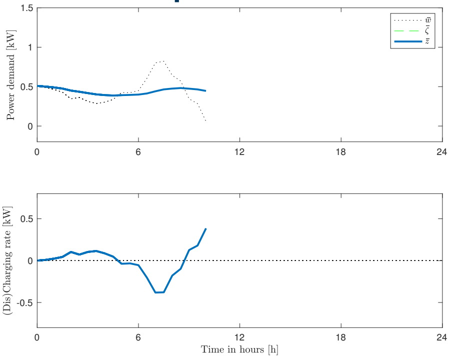

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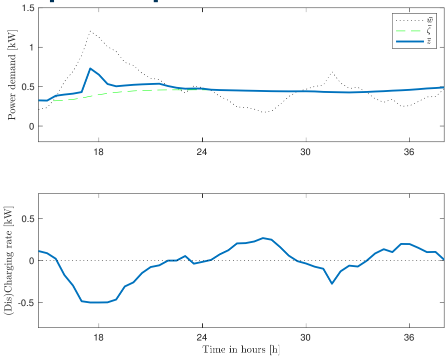

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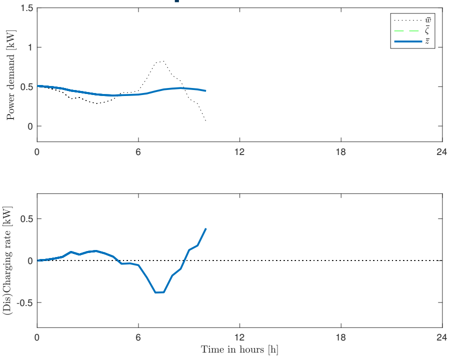

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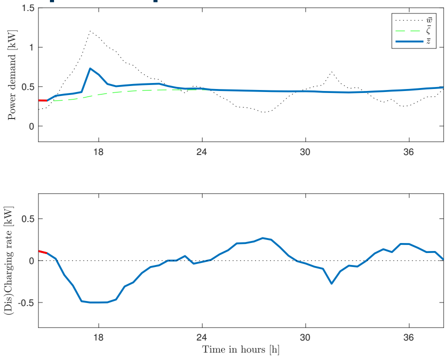

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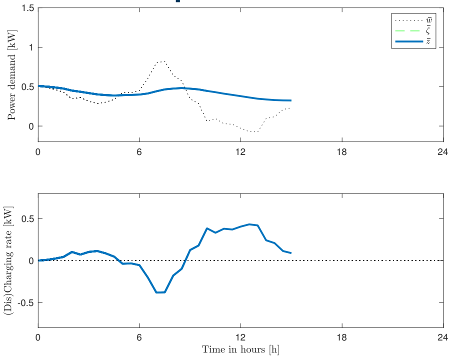

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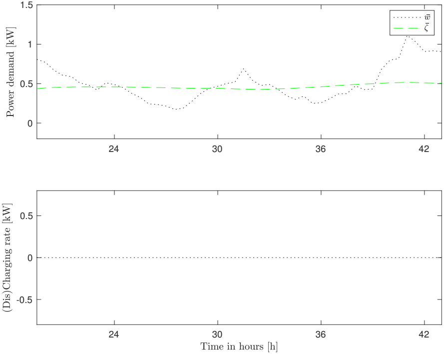

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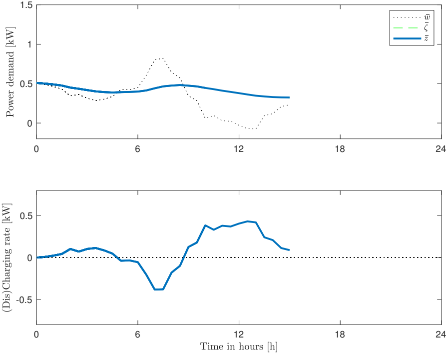

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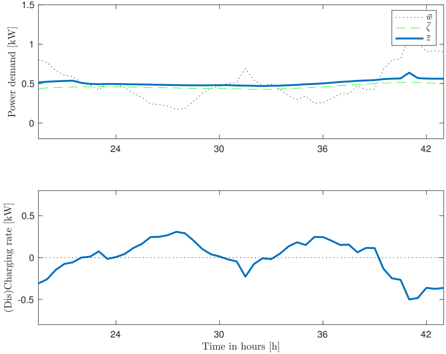

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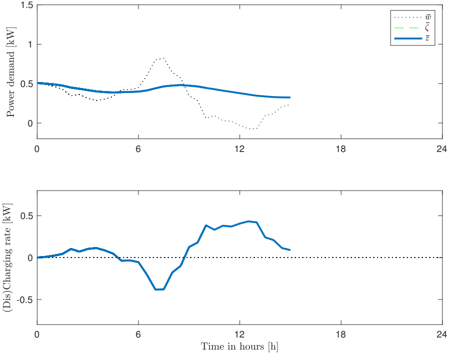

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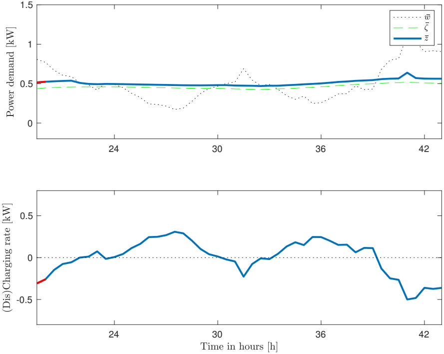

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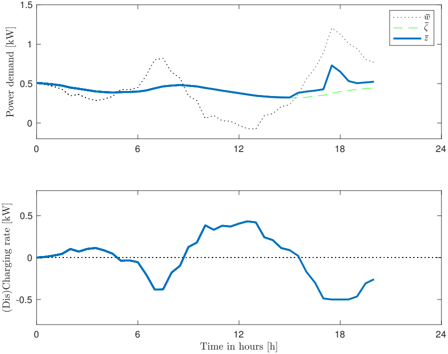

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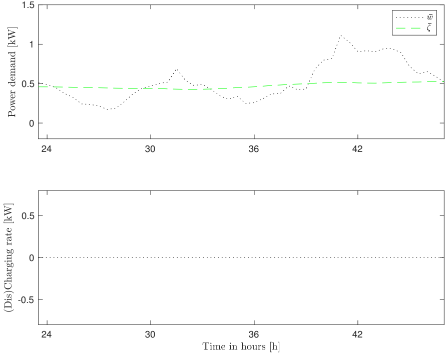

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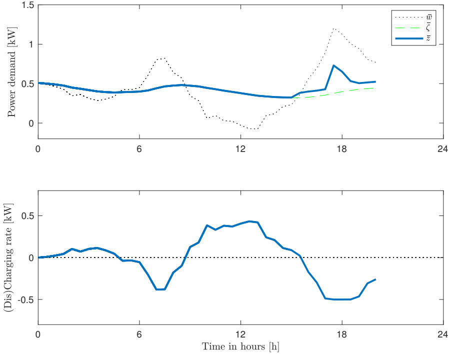

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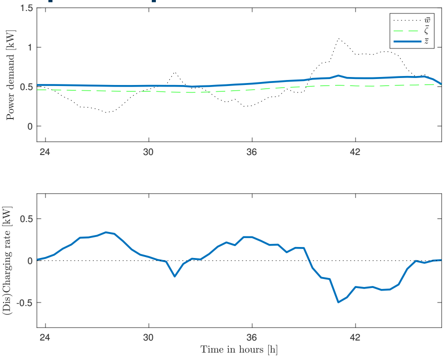

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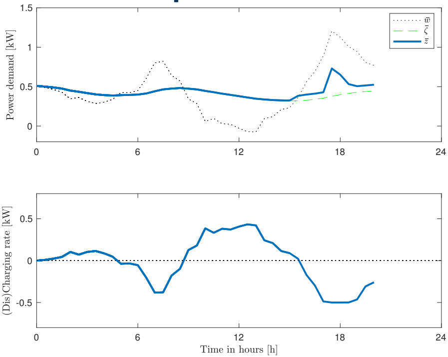

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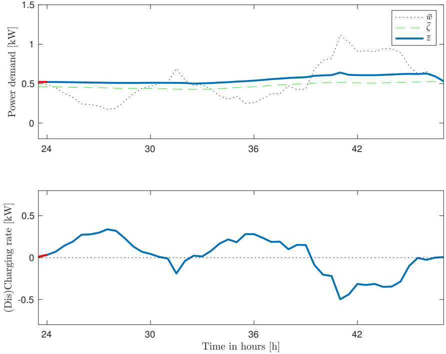

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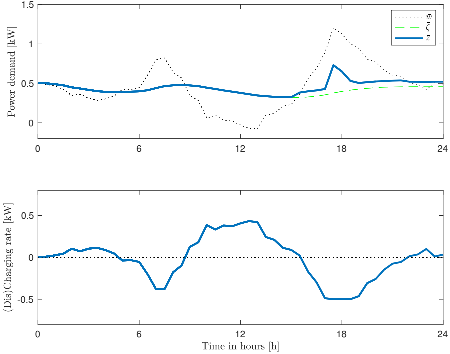

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Optimization Problem

Centralized formulation

$$\min_{u=(u^+,u^-)} \quad \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \left[w_i(n) + u_i^+(n) + \gamma_i u_i^-(n) \right] - \bar{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

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Distributed formulation

$$\min_{(z,\bar{a})} \quad \sum_{n=k}^{k+N-1} \left(\bar{a}(n) - \bar{\zeta}(n) \right)^2$$

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$$z_i - a_i = 0, i \in [1 : \mathcal{I}], \quad \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} a_i - \bar{a} = 0$$

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$$\min_{(z,\bar{a})} \sum_{n=k}^{k+N-1} \left(\bar{a}(n) - \bar{\zeta}(n) \right)^2 = \|\bar{a} - \bar{\zeta}\|_2^2 =: g(\bar{a})$$

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Alternating Direction Method of Multipliers

[Boyd et al., 2011]

Augmented Lagrangian: dual λ and $\rho > 0$

$$\mathcal{L}_\rho(z, a, \lambda) = g(\bar{a}) + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (z_i - a_i) + \frac{\rho}{2} \|z_i - a_i\|_2^2 \right)$$

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   $z_i^{\ell+1} \; = \; \arg\min_{z_i \in \mathbb{D}_i} \; z_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| z_i - a_i^\ell \right\|_2^2$

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Modified ADMM Scheme [Braun et al., 2018]

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4. Broadcast variable
   $\Pi^{\ell+1} \;=\; \bar{z}^{\ell+1} - \bar{a}^{\ell+1} + \frac{\bar{\lambda}^{\ell+1}}{\rho}$

Current Research

Distributed Optimization using ALADIN

(joint work with Yuning Jiang, ShanghaiTech University)

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Additional Local Costs

Penalize the (dis-)charging of the batteries by introducing $f_i : \mathbb{R}^{2N} \to \mathbb{R}$

$$f_i(u_i) = \|u_i\|_{Q_i}^2$$

with some scaling matrix $Q_i \in \mathbb{R}^{2N}$.

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Optimization Problem

$$\min_{\bar{z},u} \quad g(\bar{z}) + \sum_{i=1}^{\mathcal{I}} f_i(u_i)$$

$$\text{s.t.} \quad \bar{z} = \bar{w} + \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} I_N \otimes \begin{pmatrix} 1 & \gamma_i \end{pmatrix} u_i$$

$$u_i \in \mathbb{D}_i = \left\{ u_i \in \mathbb{R}^{2N} \;\middle|\; D_i u_i \le d_i \right\}, \quad i \in [1 : \mathcal{I}]$$

Distributed Optimization using ALADIN

[Houska et al., 2016]

1. Parallel step: Solve

$$\min_{v_i \in \mathbb{D}_i} \quad f_i(v_i) - \lambda^\top A_i v_i + \frac{1}{2} \|v_i - u_i\|_{H_i}^2$$

and compute $g_i = A_i^\top \lambda + H_i(u_i - v_i)$ in parallel.

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3. Consensus step: Update μ and $H_i = 2Q_i + \mu D_i^{\mathrm{act}\top} D_i^{\mathrm{act}}$ optionally, solve

$$\min_{\bar{z}^+, u^+} \quad g(\bar{z}^+) + \sum_{i=1}^{\mathcal{I}} \frac{1}{2} \|u_i^+ - v_i\|_{H_i}^2 + g_i^\top u_i^+$$

$$\text{s.t.} \quad \bar{z}^+ = \bar{w} + \sum_{i=1}^{\mathcal{I}} A_i u_i^+ \quad |\lambda^+$$

   and update $\bar{z} = \bar{z}^+$, $u = u^+$, $\lambda = \lambda^+$.

Comparison of Both Algorithms

	ADMM	ALADIN
requirements		
convergence rate		
communication $\uparrow$		
communication $\downarrow$		

Comparison of Both Algorithms

	ADMM	ALADIN
requirements	convex objective functions	
convergence rate		
communication ↑		
communication ↓		

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communication ↑	N	
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Outlook: ALADIN could be applied to

- more complex (in particular non-convex) objective functions
- nonlinear battery dynamics

Multiobjective Optimization Problem
[S., Worthmann, 2019]

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Multiobjective Optimization Problem Formulation

$$\min_{\bar{z},s} \quad \begin{pmatrix} g(\bar{z}) \\ h(s) \end{pmatrix} \quad \text{s.t.} \quad \text{system dynamics and constraints and (1)}$$

Efficiency, Pareto Frontier

Definition

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$$\min_{(\bar{z},s)\in\mathbb{S}} \begin{pmatrix} g(\bar{z}) \\ h(s) \end{pmatrix}, \tag{2}$$

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The set

$$\left\{ (g(\bar{z}^*), h(s^*)) \in \mathbb{R}^2 \;\middle|\; (\bar{z}^*, s^*) \text{ is efficient for } (2) \right\}$$

is called Pareto frontier.

Characterization of Efficiency

Proposition (S., Worthmann, 2019, Ehrgott, 2005)

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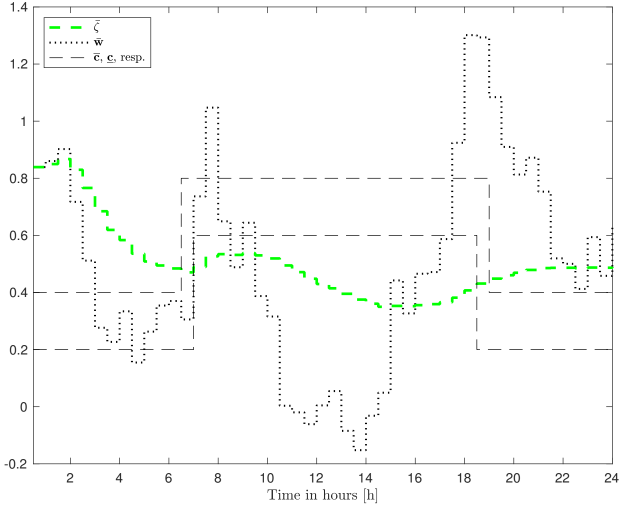
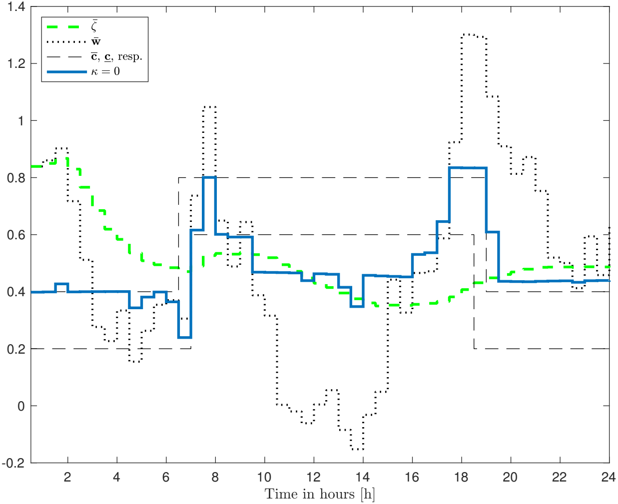
Efficient Control

Setting



Efficient Control

Open Loop Performance



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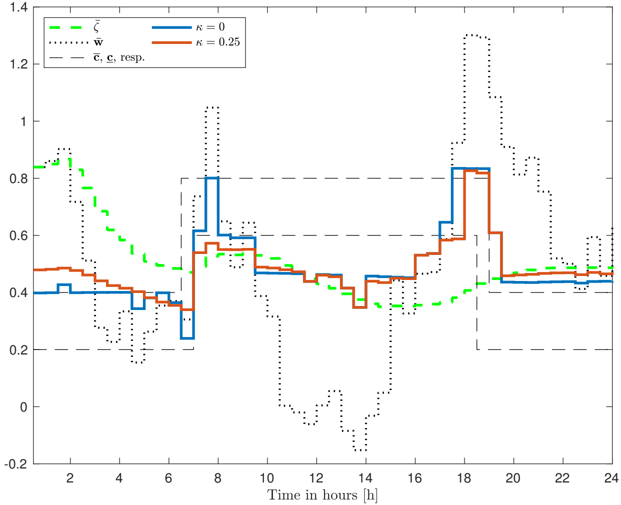

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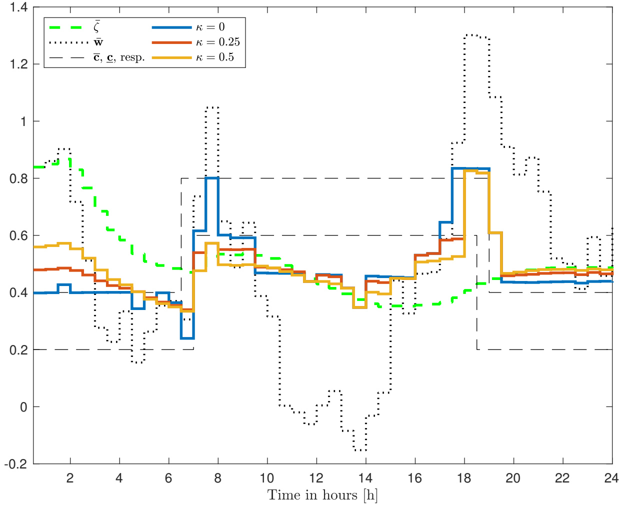

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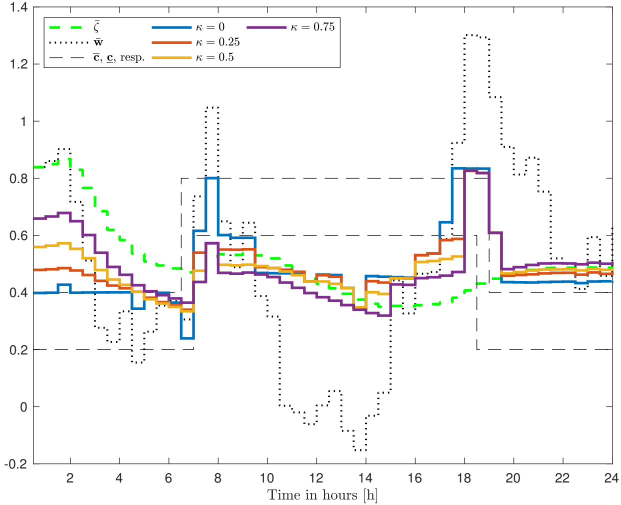

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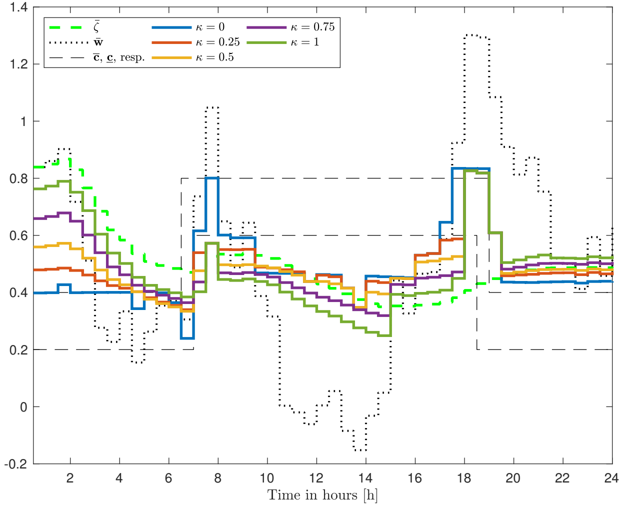

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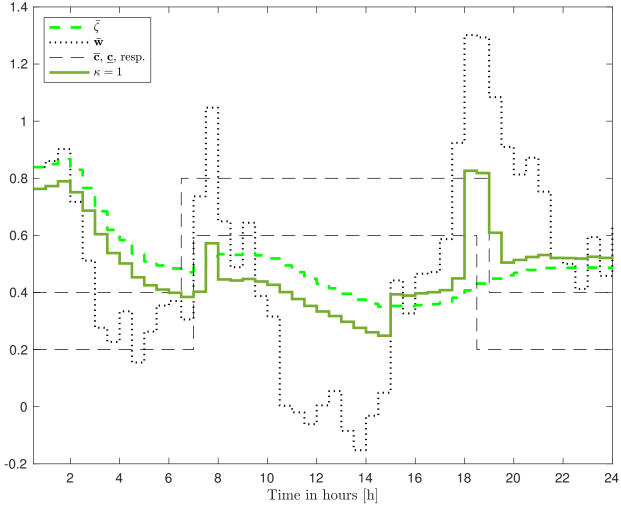
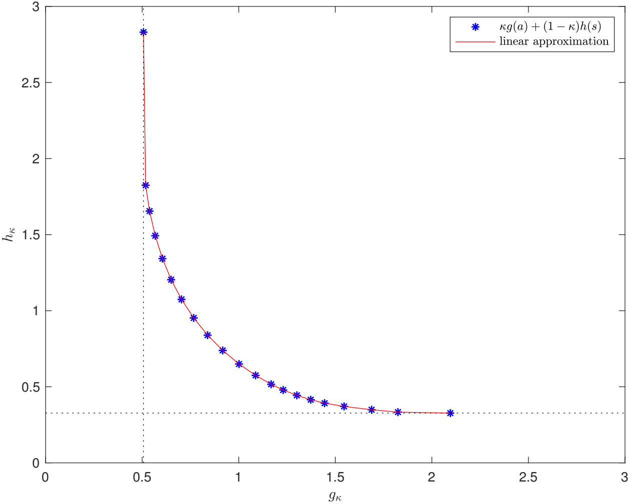


Efficient Control

Pareto Frontier



Proper Efficiency

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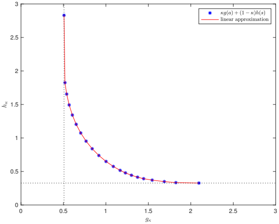

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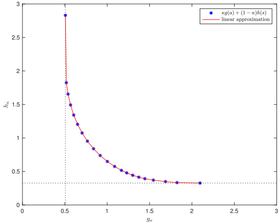

Interpretation

- efficient points = non-dominated points

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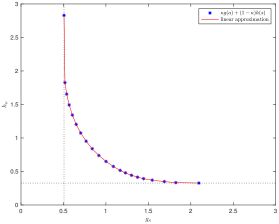

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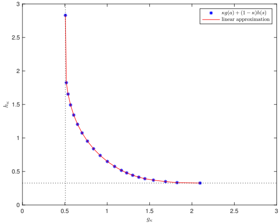

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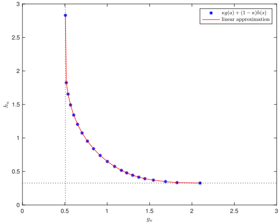

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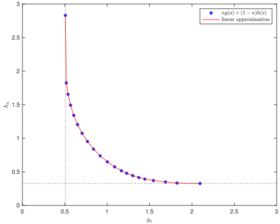

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$\rightsquigarrow$ information on costs of improvement w.r.t. one objective

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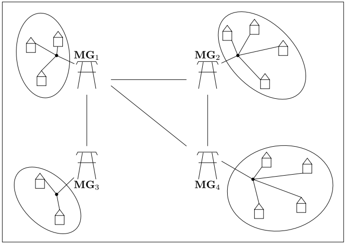

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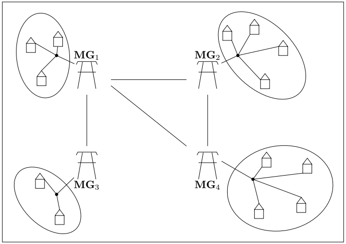

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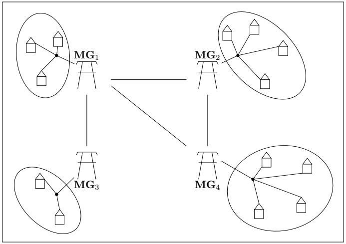

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3	32.25	79.82
4	34.03	85.07
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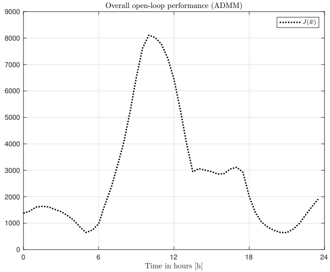

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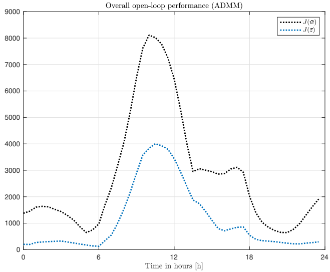
Results

MG	$\Vert\bar{z} - \bar{\zeta}\Vert_2^2$	$\Vert\bar{z}^+ - \bar{\zeta}\Vert_2^2$
1	920.57	451.44
2	124.85	85.07
3	32.25	79.82
4	34.03	85.07
overall	1111.70	701.39



Bilevel Optimization Scheme

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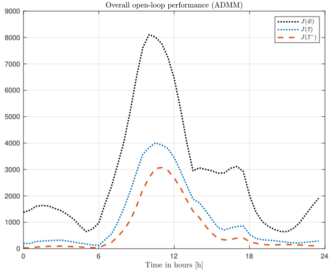

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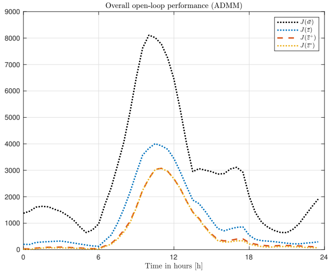

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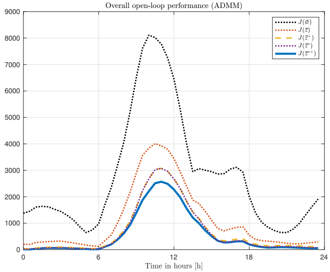

Surrogate Models for Microgrids

(joint work with Sara Grundel and Manuel Baumann, MPI Magdeburg, [Grundel, S., Worthmann, 2019])

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Approach for single MGs

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$\rightsquigarrow$ solve optimization problem in each MPC iteration

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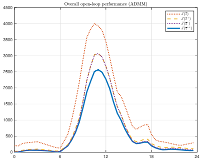

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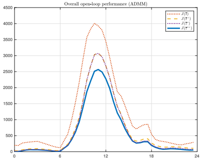

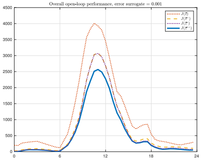

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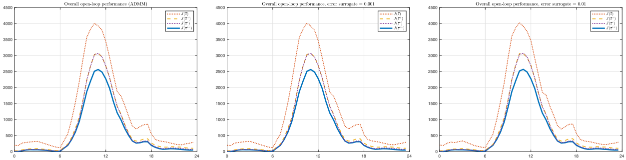

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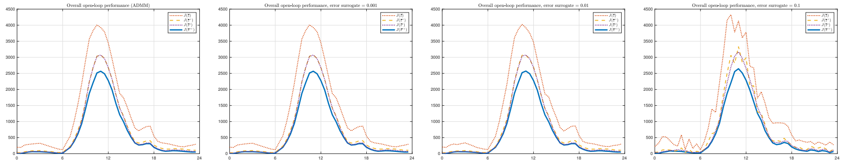
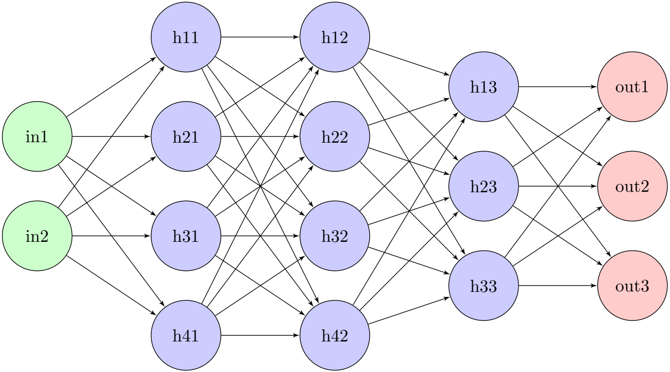


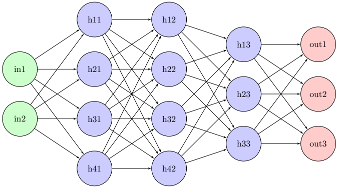
Introduction to Neural Networks

Basic Scheme incorporating 5 layers



- 1 input layer with 2 neurons
- 1 output layer with 3 neurons
- 3 hidden layers with 3 or 4 neurons, resp.

Introduction to Neural Networks



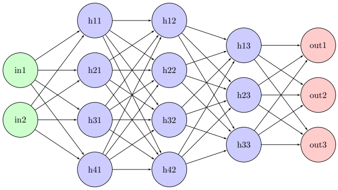
- sigmoid function $\sigma(x) = \frac{1}{1+\mathrm{e}^{-x}}$
- weights $W^{[\ell+1]} \in \mathbb{R}^{m_{\ell+1} \times m_\ell}$
- biases $b^{[\ell+1]} \in \mathbb{R}^{m_{\ell+1}}$

with number m_ℓ of neutrons in layer ℓ.

Output of layer $\ell + 1$ given output y^ℓ of previous layer

$$y^{\ell+1} = \sigma(W^{[\ell+1]} y^\ell + b^{[\ell+1]})$$

Introduction to Neural Networks



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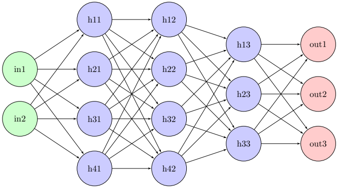
Output of neural network given input x

$$z = F(x) = \sigma(W^{[4]}\sigma(W^{[3]}\sigma(W^{[2]}x + b^{[2]}) + b^{[3]}) + b^{[4]})$$

Introduction to Neural Networks



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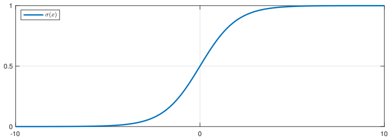
Sigmoid function



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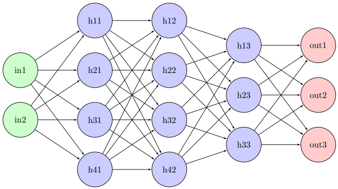

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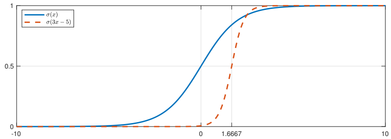

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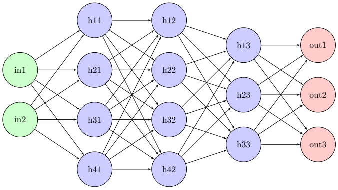

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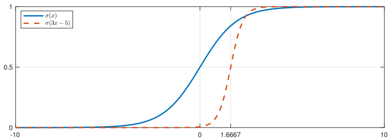
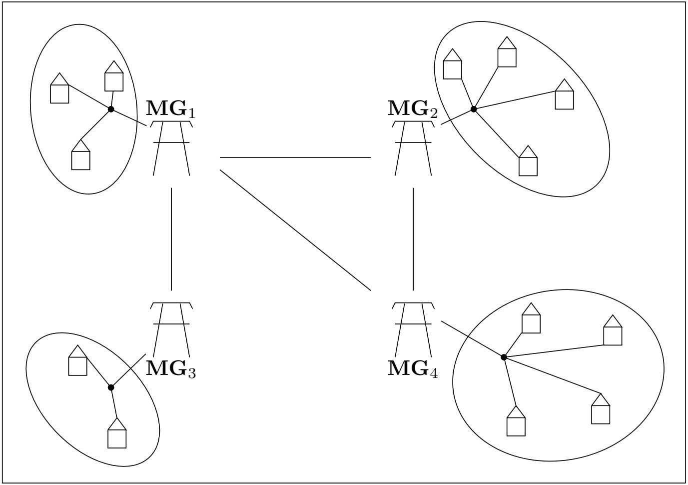


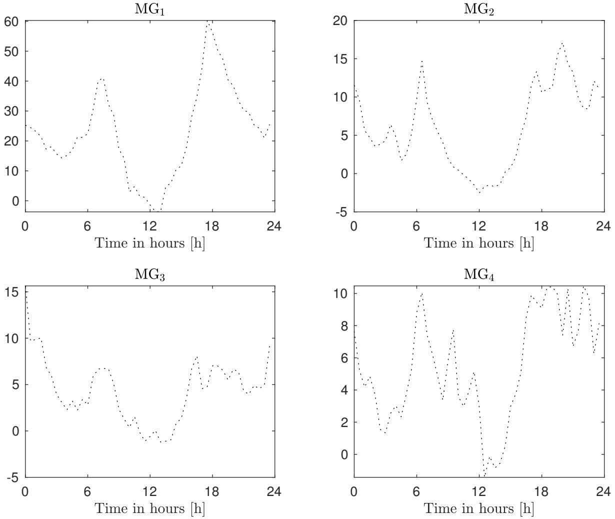
- "smoothed" step function
- imitating neurons in brain:
  - $\sigma = 1 \Leftrightarrow$ neuron firing
  - $\sigma = 0 \Leftrightarrow$ neuron inactive
- [Higham, Higham, 2018]

Results

Coupled MGs

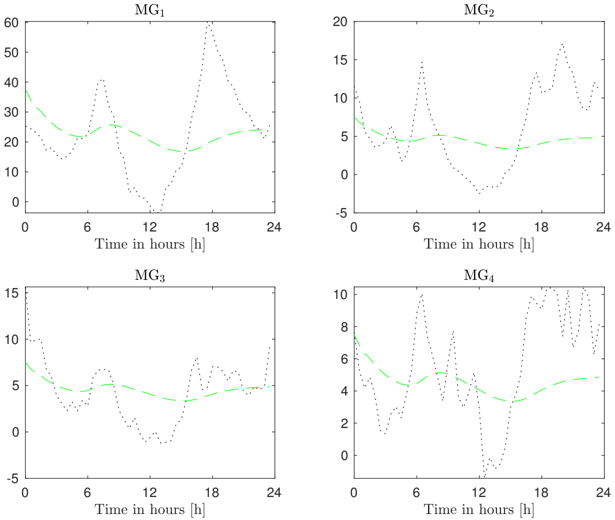


Results
Net consumption



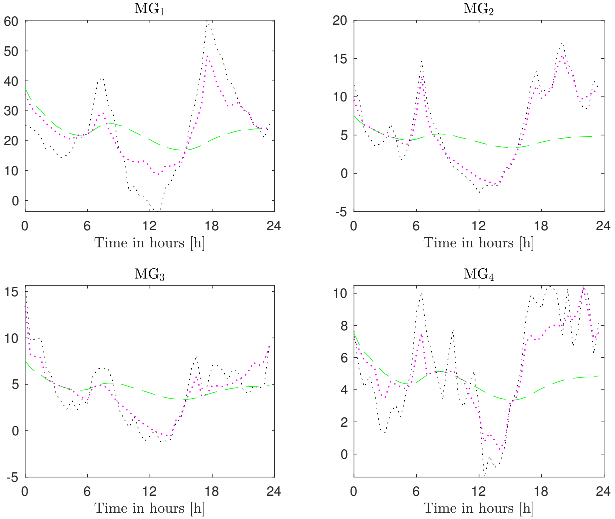
Results

Reference trajectory



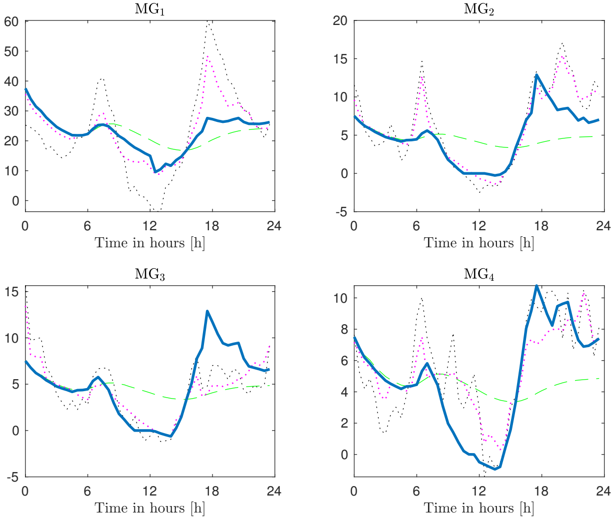
Results

Solution without additional exchange



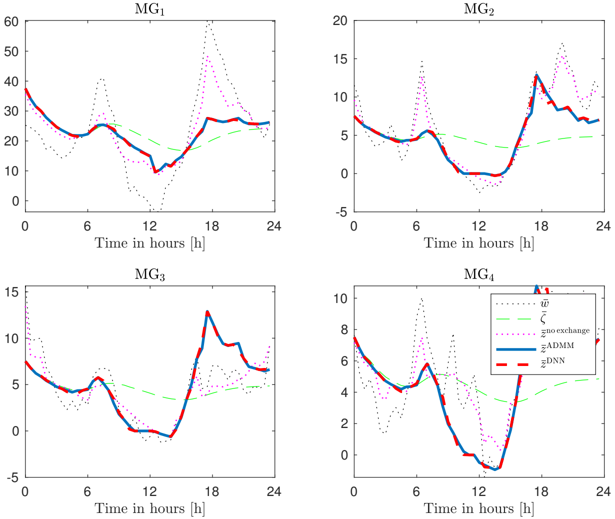
Results

Solution incorporating exchange within MGs



Results

Approximation (using MatLab's Neural Network)



Thank you for your attention!

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