

# How to Coordinate Countermeasures against COVID-19

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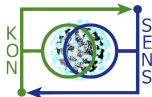
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Federal Ministry of Education and Research

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Federal Ministry  
of Education  
and Research

ECMI, 14 April 2021



# Outline

## 1. Epidemiological modelling

1.1 Basics: S(E)IR model

1.2 Extensions tailored to COVID-19: SEIPHR model

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## 2. Countermeasures

2.1 Social distancing

2.2 Vaccination

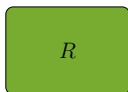
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- 1. Epidemiological modelling**
  - 1.1 Basics: S(E)IR model
  - 1.2 Extensions tailored to COVID-19: SEIPHR model
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- 3. Long-term vs. short-term optimization**
  - 3.1 Model predictive control
  - 3.2 Results

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  - 3.2 Results
- 4. Conclusions & outlook**

# Basic compartmental model: SIR



## 3 compartments

- susceptible  $S$
- infectious  $I$
- removed  $R$  (recovered/deceased)

# Basic compartmental model: SIR

$\beta$  :  $\frac{\text{average number of contacts}}{\text{person} \times \text{time}} \cdot \text{Prob}(\text{transmission})$



## 3 compartments

- susceptible  $S$
- infectious  $I$
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$$\frac{d}{dt}S(t) = -\beta S(t)I(t)$$

$$\frac{d}{dt}I(t) = \beta S(t)I(t)$$

# Basic compartmental model: SIR

$\eta^{-1}$  : average recovery time



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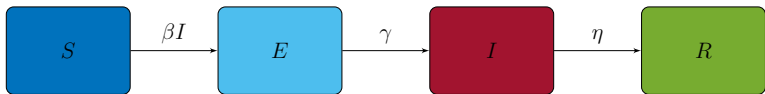
$$\frac{d}{dt}I(t) = \beta S(t)I(t) - \eta I(t)$$

$$\frac{d}{dt}R(t) = \eta I(t)$$



# Basic compartmental model: SEIR

$\gamma^{-1}$  : average incubation time



## 4 compartments

- susceptible  $S$
- exposed  $E$
- infectious  $I$
- removed  $R$  (recovered/deceased)

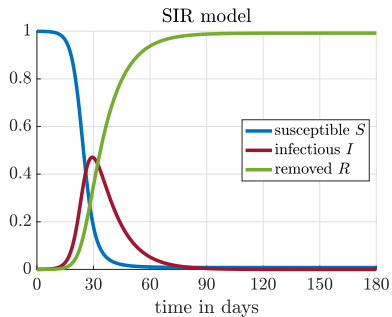
$$\frac{d}{dt}S(t) = -\beta S(t)I(t)$$

$$\frac{d}{dt}E(t) = \beta S(t)I(t) - \gamma E(t)$$

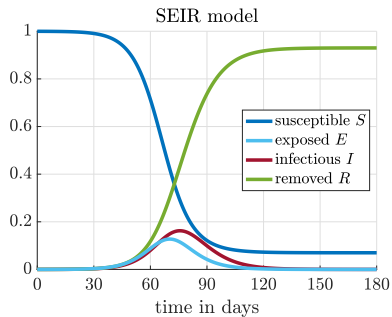
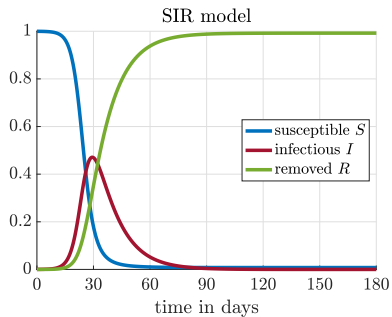
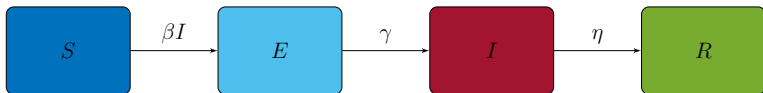
$$\frac{d}{dt}I(t) = \gamma E(t) - \eta I(t)$$

$$\frac{d}{dt}R(t) = \eta I(t)$$

# Basic compartmental model: SEIR



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  - social distancing/quarantine
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  - mass testing

# Drawbacks of the SEIR model

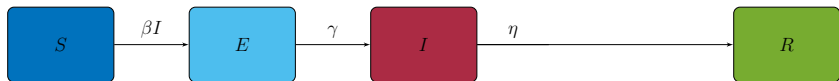
## Model does not account for

- symptom severity (ICU occupancy)
- demographic influences
  - on contacts
  - symptom severity
- counter measures
  - social distancing/quarantine
  - vaccination
  - mass testing
- births and (natural) deaths
- re-infections



# Extension: SEIPHR model

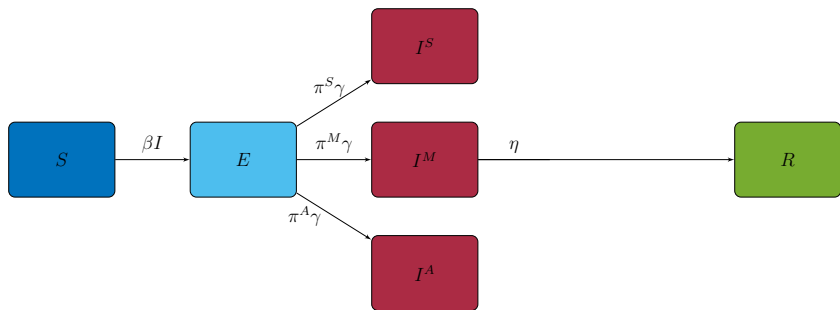
## SEIR model



# Extension: SEIPHR model

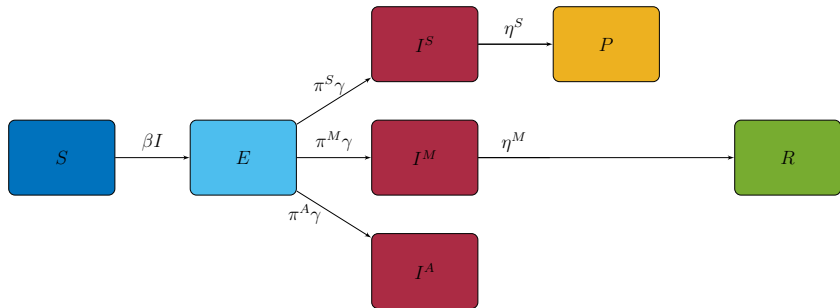
## Symptom severity

transmission probabilities  $\pi^S + \pi^M + \pi^A = 1$



# Extension: SEIPHR model

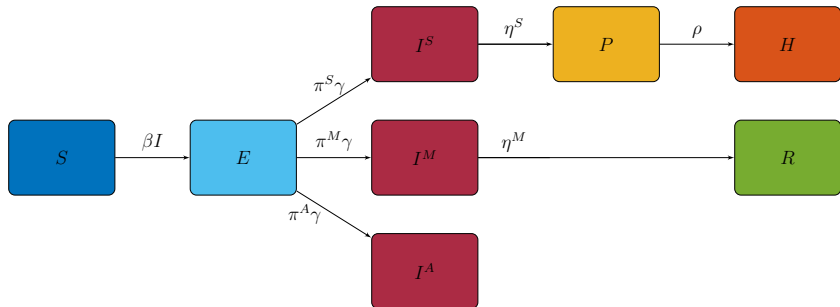
Pre-ICU compartment  
quarantine



# Extension: SEIPHR model

## ICU compartment

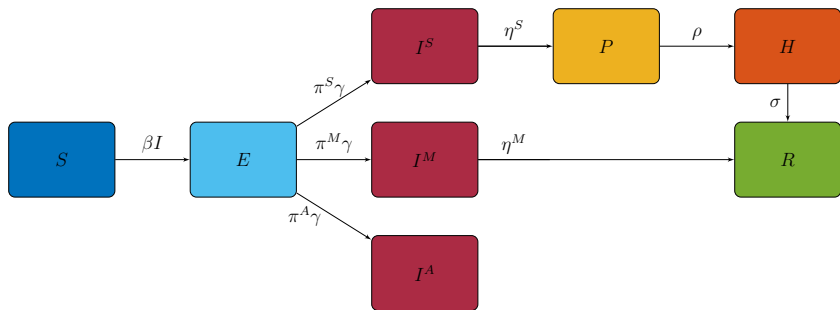
$\rho$ : ICU admittance rate



# Extension: SEIPHR model

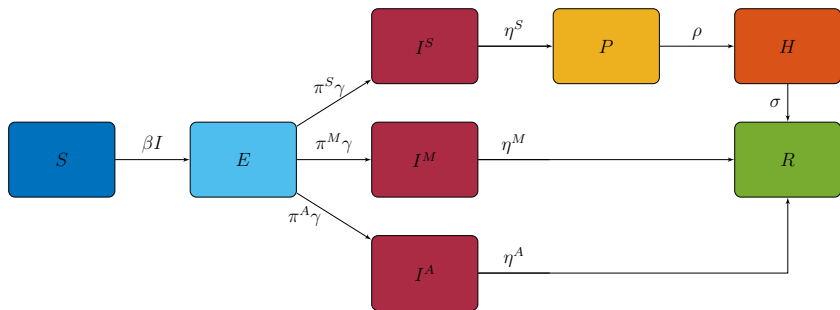
## ICU compartment

$\sigma$ : ICU discharge rate



# Extension: SEIPHR model

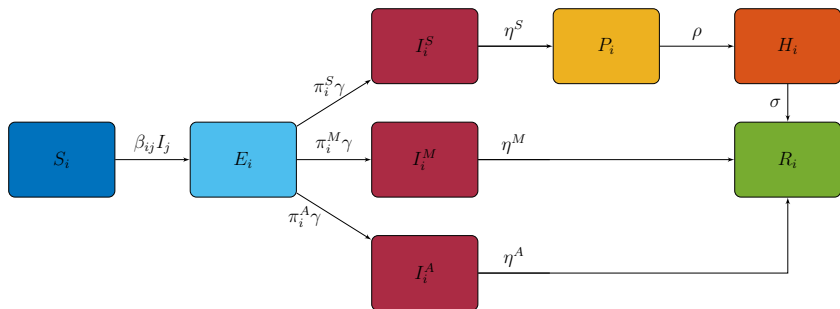
## Undetected recovery



# Extension: SEIPHR model

## Distinguish age groups

$i \in \{1, 2, \dots, n_g\}$  (in particular  $\beta_{ij}$  and  $\pi_i$ )

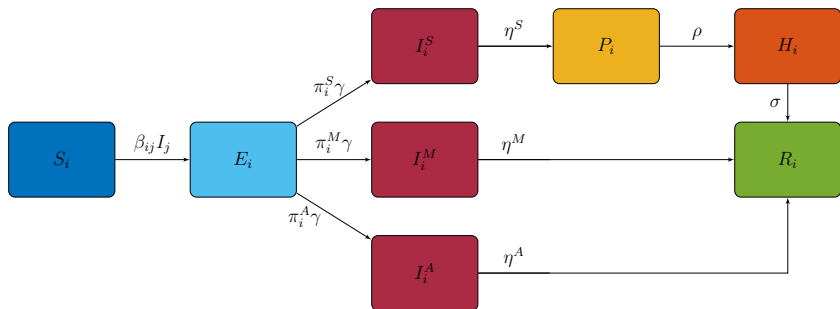


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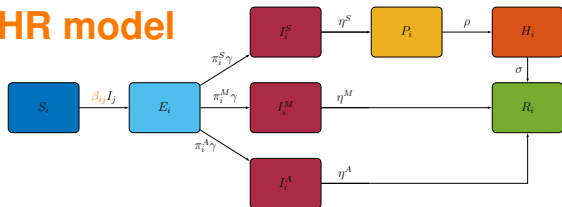
$i \in \{1, 2, \dots, n_g\}$  (in particular  $\beta_{ij}$  and  $\pi_i$ )

$n_g = 3$ : children, adults (most contacts), elderly (high-risk)





# Extension: SEIPHR model



## System dynamics

$$\frac{d}{dt} S_i(t) = - \sum_{j=1}^{n_g} \beta_{ij} S_i(t) I_j(t) \quad (I_i = I_i^S + I_i^M + I_i^A)$$

$$\frac{d}{dt} E_i(t) = \sum_{j=1}^{n_g} \beta_{ij} S_i(t) I_j(t) - \gamma E_i(t)$$

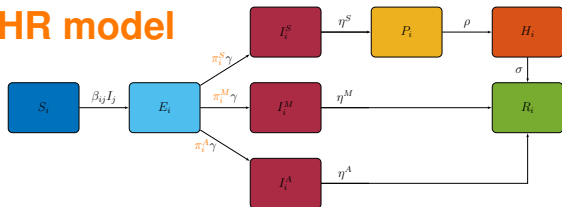
$$\frac{d}{dt} I_i^\#(t) = \pi_i^\# \gamma E_i(t) - \eta^\# I_i^\#(t) \quad \# \in \{S, M, A\}$$

$$\frac{d}{dt} P_i(t) = \eta^S I_i^S(t) - \rho P_i(t)$$

$$\frac{d}{dt} H_i(t) = \rho P_i(t) - \sigma H_i(t)$$

$$\frac{d}{dt} R_i(t) = \eta^M I_i^M(t) + \eta^A I_i^A(t)$$

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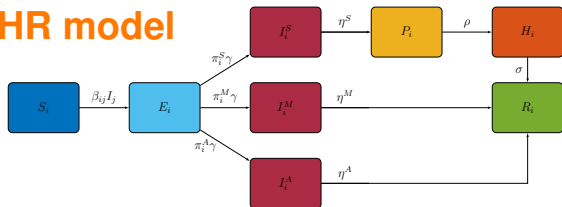
$$\frac{d}{dt} I_i^\#(t) = \pi_i^\# \gamma E_i(t) - \eta^\# I_i^\#(t) \quad \# \in \{S, M, A\}$$

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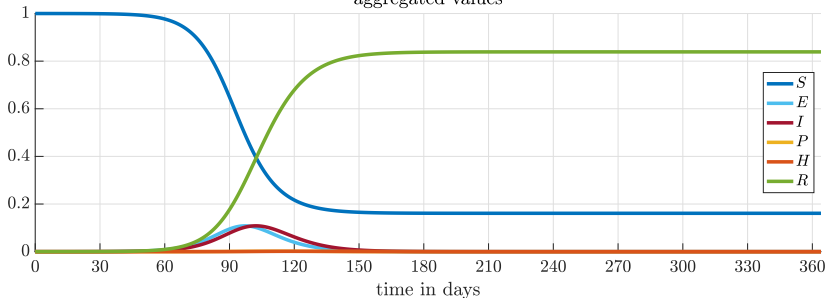
$$\frac{d}{dt} R_i(t) = \eta^M I_i^M(t) + \eta^A I_i^A(t)$$

# Extension: SEIPHR model

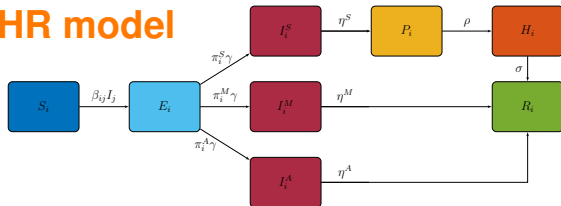


## Simulation results

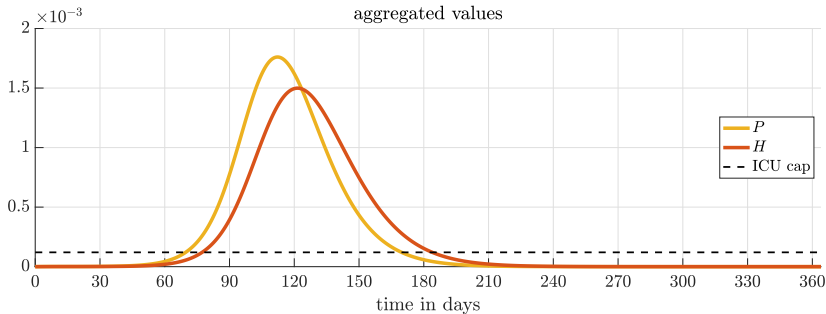
aggregated values



# Extension: SEIPHR model

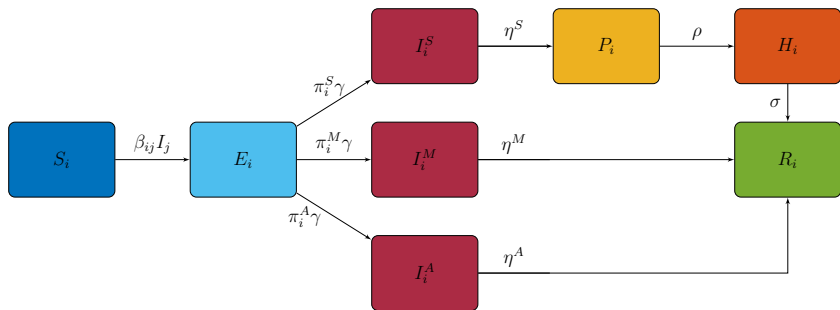


## Simulation results



# Social distancing

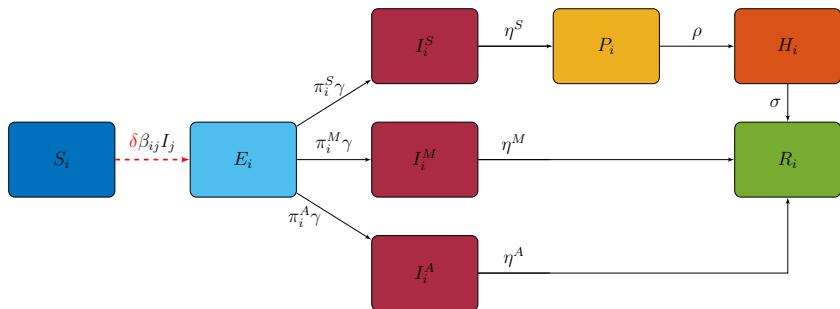
## SEIPHR model



# Social distancing

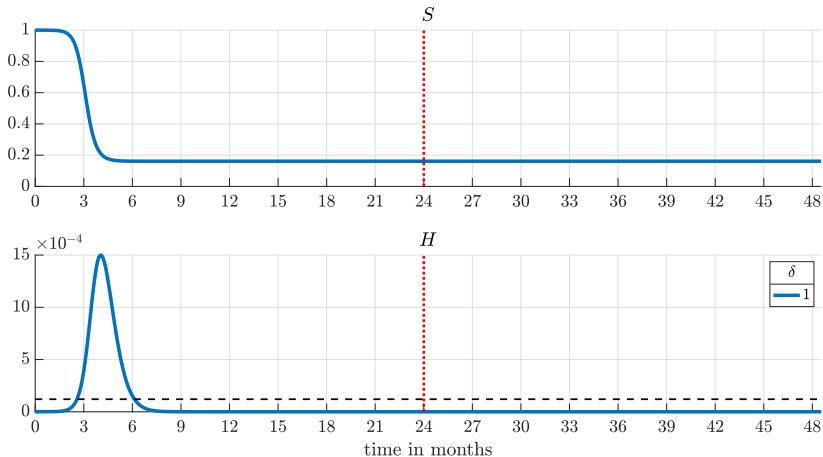
## Contact restrictions

control input  $\delta : [0, \infty) \rightarrow [0, 1]$



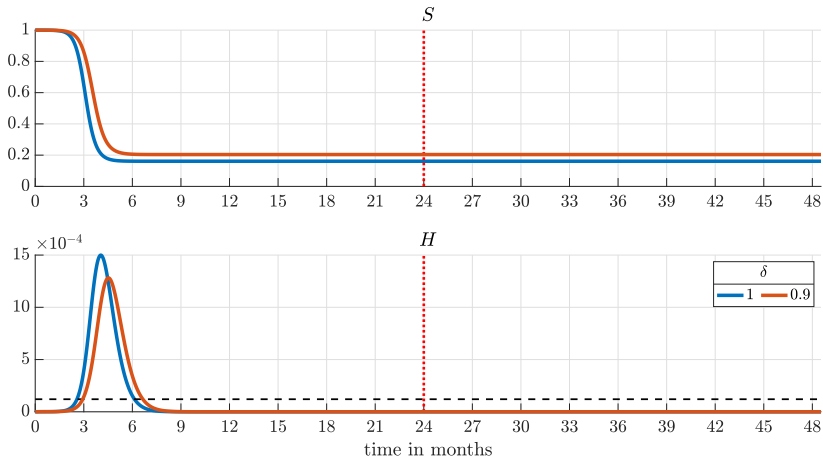
# Impact of constant restrictions

Simulation results over 4 years with lift of restrictions after 2 years



# Impact of constant restrictions

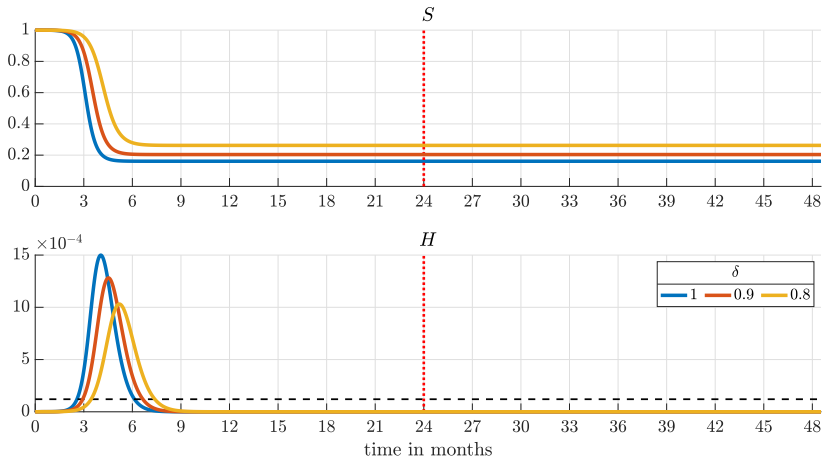
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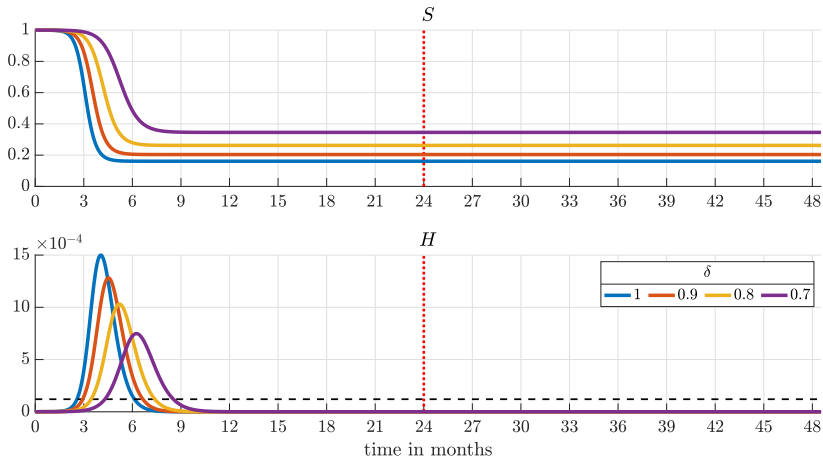
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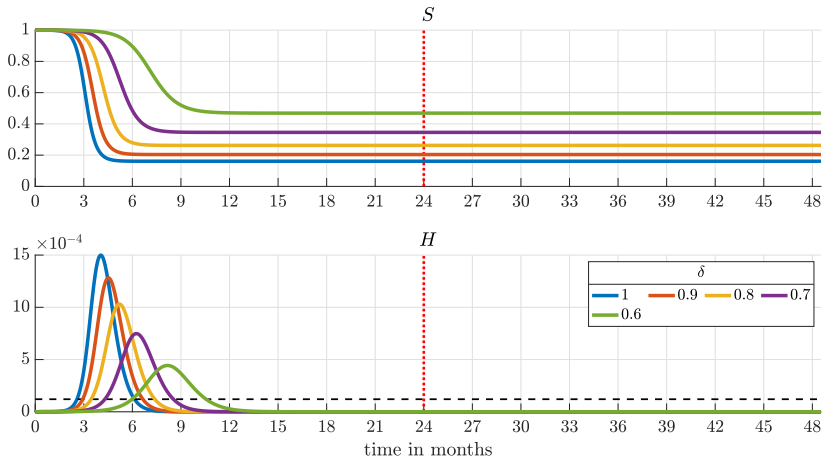
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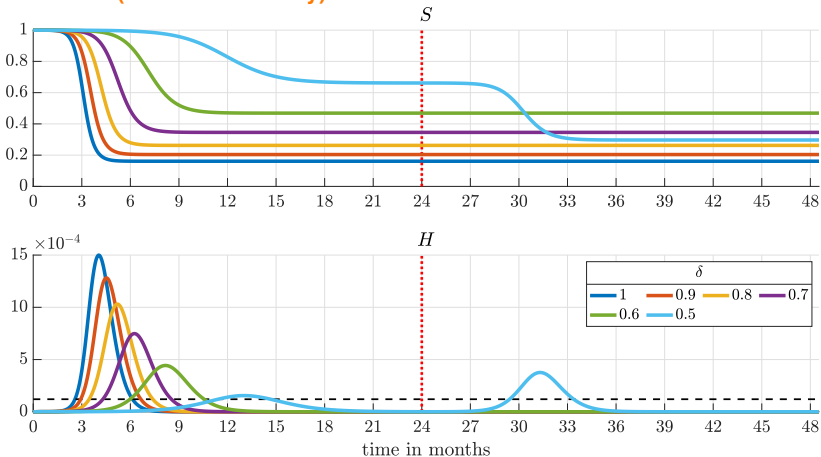
Simulation results over 4 years with lift of restrictions after 2 years



# Impact of constant restrictions

Simulation results over 4 years with lift of restrictions after 2 years:

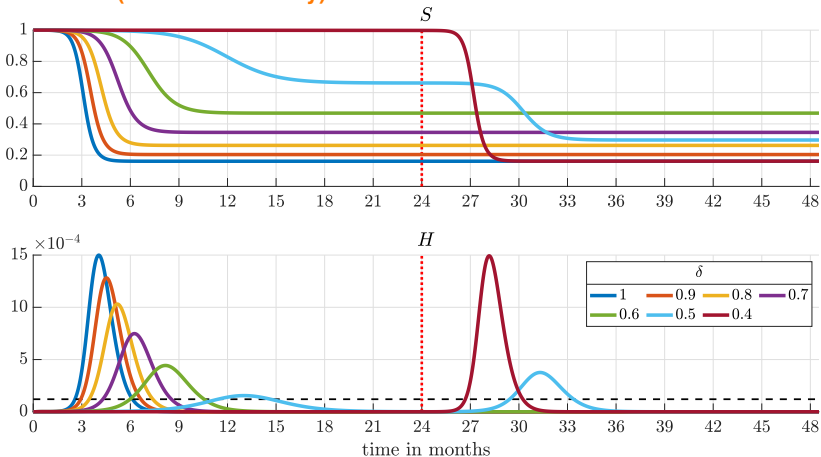
2nd wave (no herd immunity)



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# Optimal social distancing

**Goal:** maintain hard ICU cap with as few contact restrictions as possible

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## Optimal control problem

$$\begin{aligned} & \min_{\delta} \int_0^{t_f} (1 - \delta(t))^2 dt \\ \text{subject to } & \dot{x}(t) = f(x(t), \delta(t)), \quad x(0) = x^0 \\ & \sum_{i=1}^{n_g} H_i(t) \leq H^{\max} \quad \forall t \geq 0 \\ & \delta(t) \in [0, 1] \quad \forall t \geq 0 \end{aligned}$$

# Optimal social distancing

**Goal:** maintain hard ICU cap with as few contact restrictions as possible

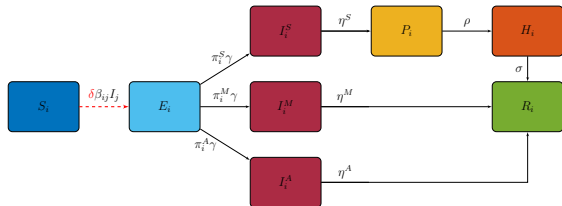
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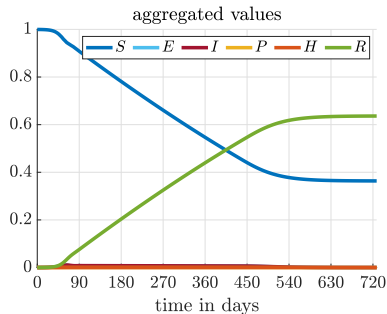
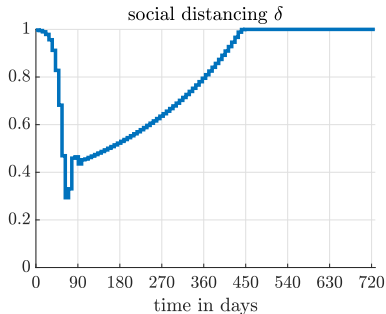
with  $\Delta t = 1$  week



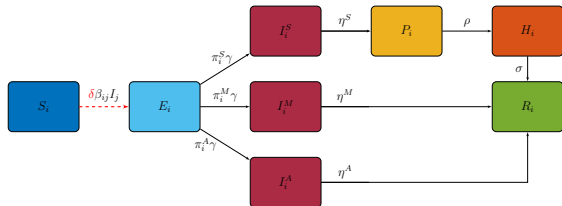
# Results



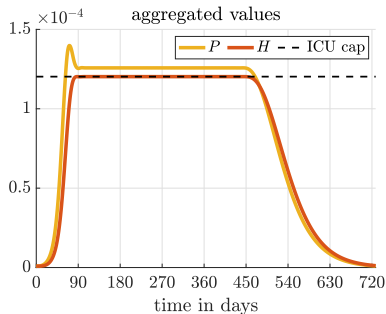
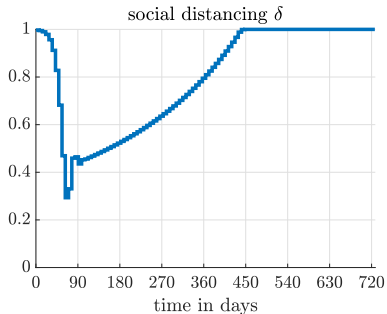
## Simulation results



# Results



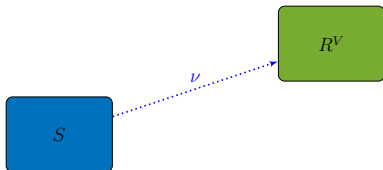
## Simulation results



# Vaccination

## Vaccination of susceptible people

vaccination rate  $\nu : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$

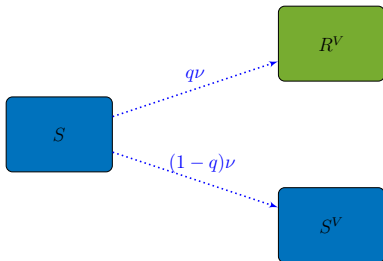


# Vaccination

## Vaccination of susceptible people

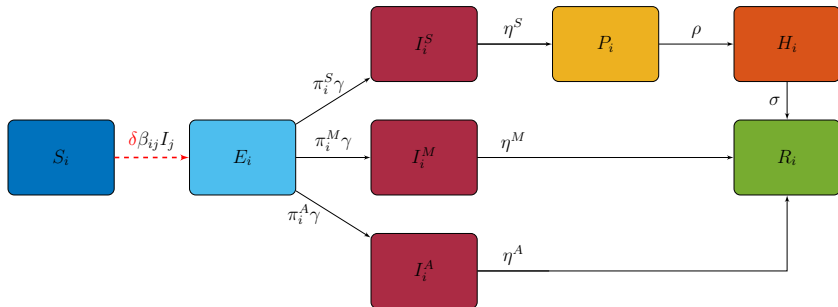
vaccination rate  $\nu : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$

success rate  $q \in [0, 1]$



# Vaccination

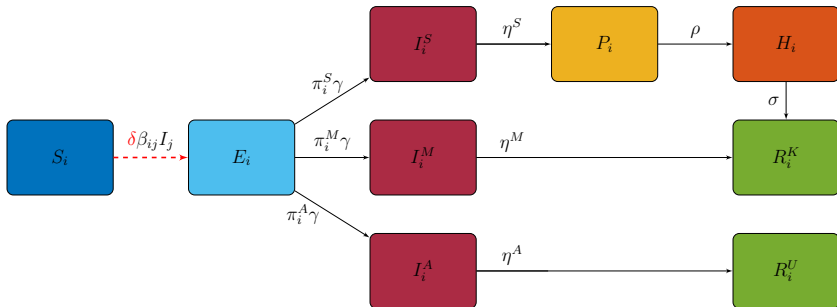
## SEIPHR model



# Vaccination

## SEIPHR model

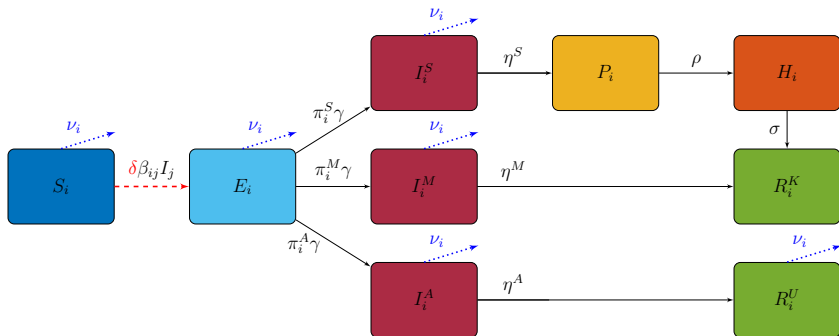
separate undetected recovery



# Vaccination

## Non-vaccinated part of population

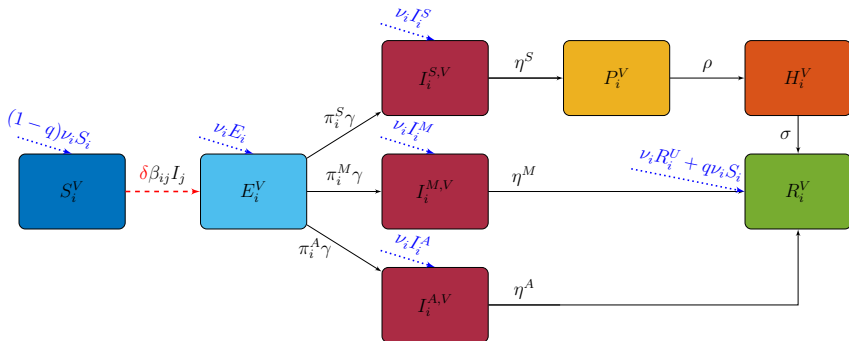
vaccination rate  $\nu_i : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$



# Vaccination

## Vaccinated part of population

vaccination rate  $\nu_i : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$





# Coordination of social distancing & vaccination

Goal: reduce social distancing

Optimal control problem

$$\begin{aligned} \min_{\delta} \quad & \int_0^{t_f} (1 - \delta(t))^2 dt \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), \delta(t)), \quad x(0) = x^0 \\ & \sum_{i=1}^{n_g} H_i(t) \leq H^{\max} \quad \forall t \geq 0 \\ & \delta(t) \in [0, 1] \quad \forall t \geq 0 \\ & \delta(t) = \delta(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, 1, \dots \end{aligned}$$

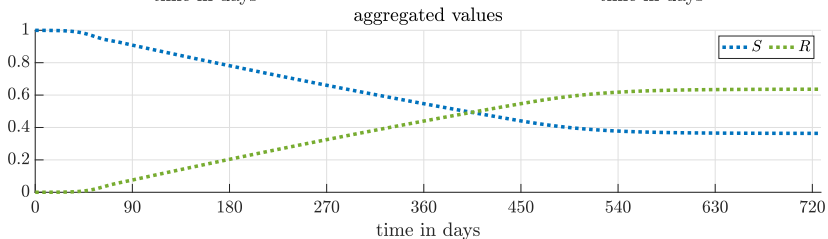
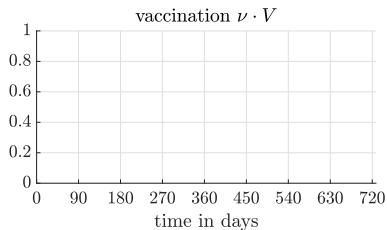
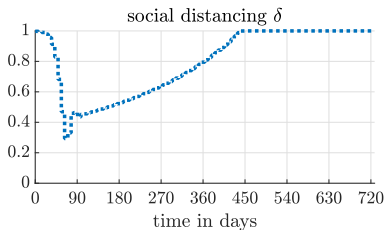
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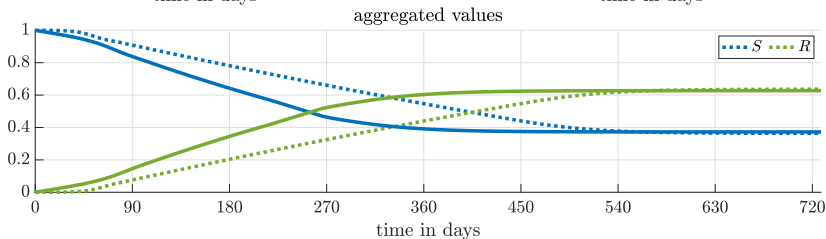
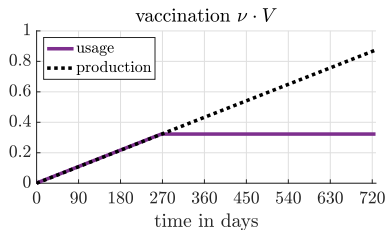
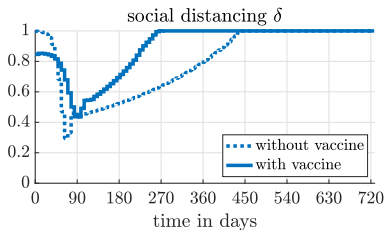
Optimal control problem

$$\begin{aligned} \min_{\delta, \nu} \quad & \int_0^{t_f} (1 - \delta(t))^2 dt + \kappa \|\nu\|_2^2 \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), \delta(t), \nu(t)), \quad x(0) = x^0 \\ & \sum_{i=1}^{n_g} H_i(t) + H_i^V(t) \leq H^{\max} \quad \forall t \geq 0 \\ & \delta(t) \in [0, 1] \quad \forall t \geq 0 \\ & \delta(t) = \delta(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, 1, \dots \\ & \int_0^t \sum_{i=1}^{n_g} \nu_i(s) V_i(s) ds \leq V^{\max} \cdot t \quad \forall t \geq 0 \\ & \nu(t) = \nu(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, 1, \dots \end{aligned}$$

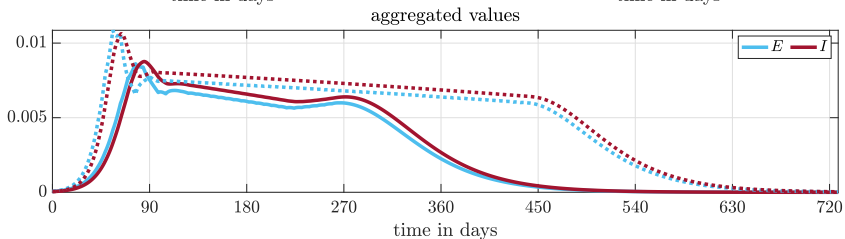
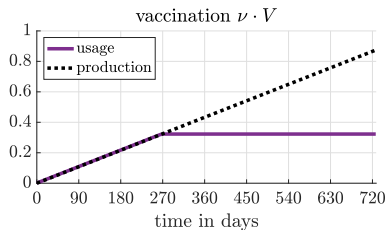
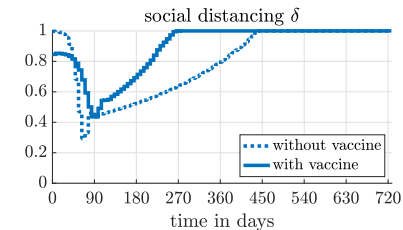
# Results



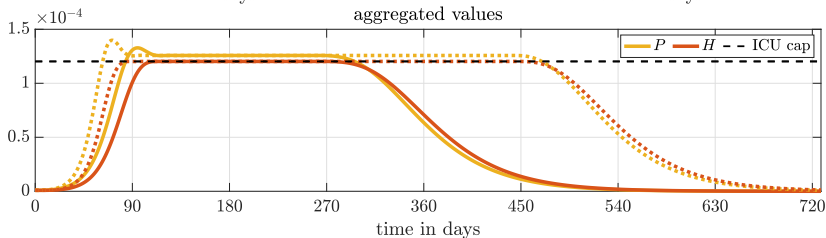
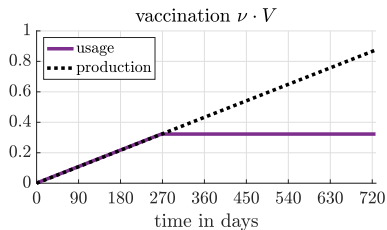
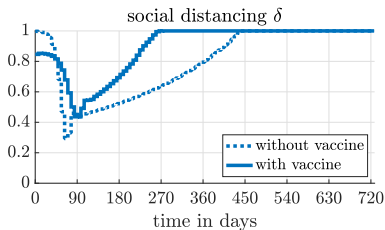
# Results



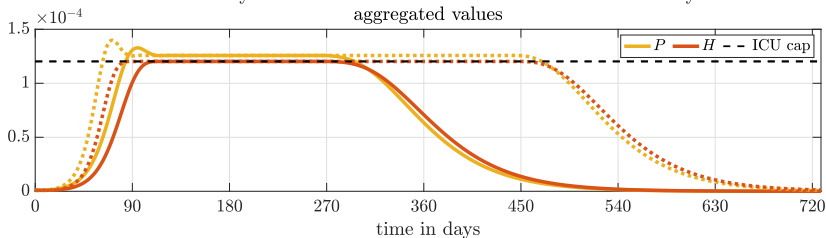
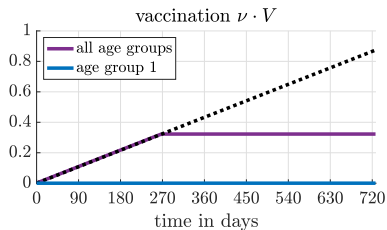
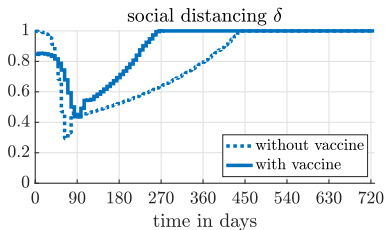
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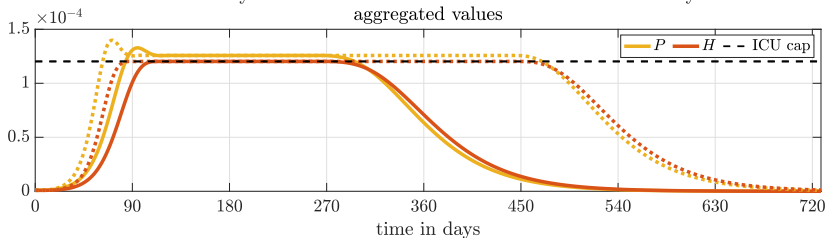
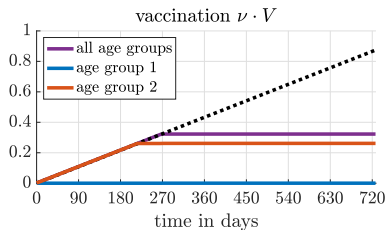
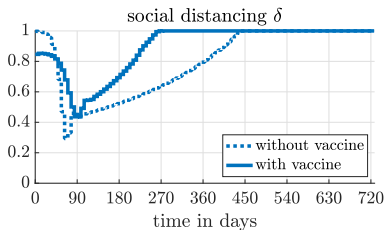
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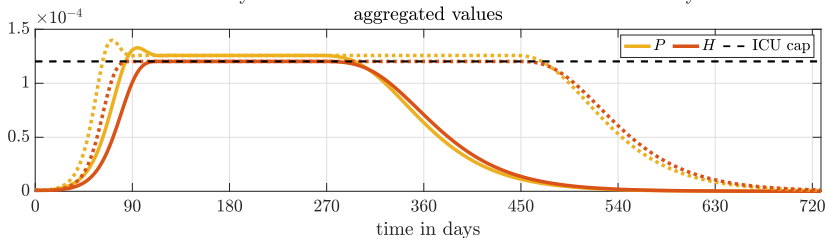
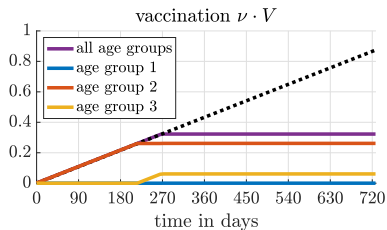
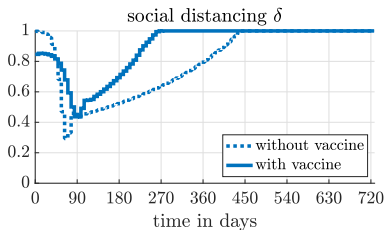


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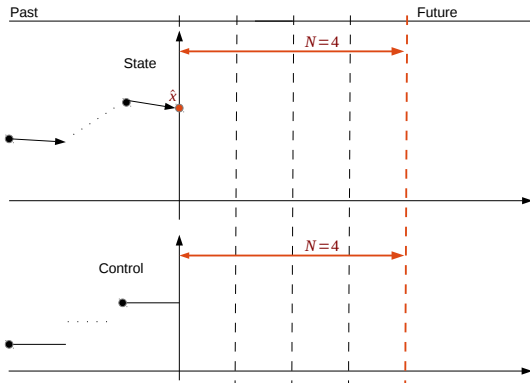




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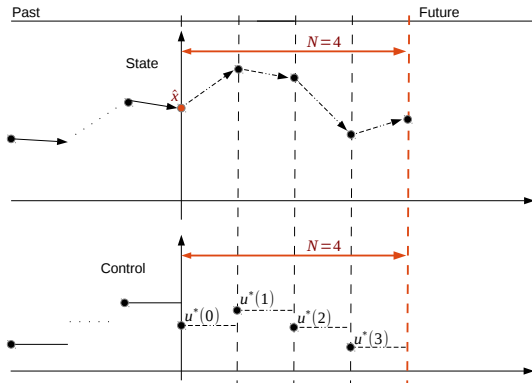


# Model Predictive Control



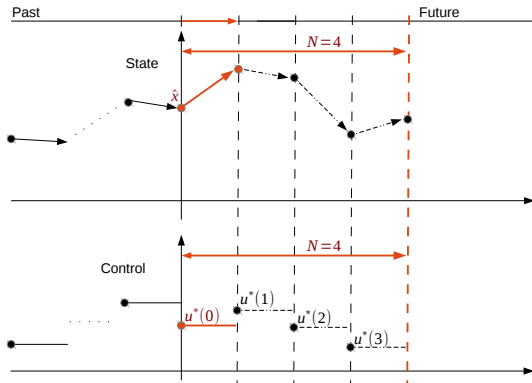
1. Measure/Estimate current state and update parameters.

# Model Predictive Control



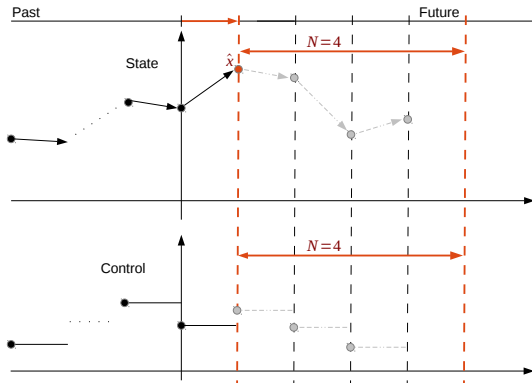
1. Measure/Estimate current state and update parameters.
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# Model Predictive Control



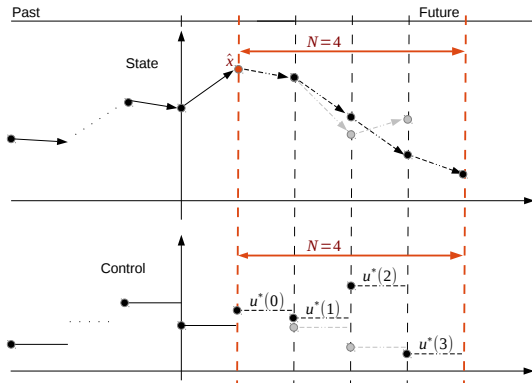
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3. Implement first control instance.

# Model Predictive Control



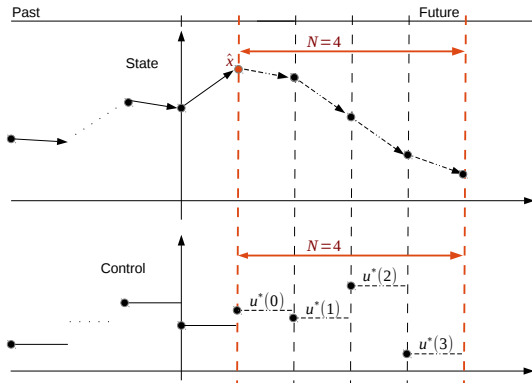
1. Shift time step, measure/estimate current state, and update parameters.
2. Solve optimal control problem *on small time window*.
3. Implement first control instance.

# Model Predictive Control



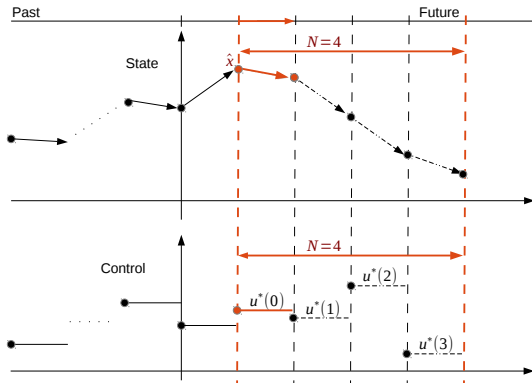
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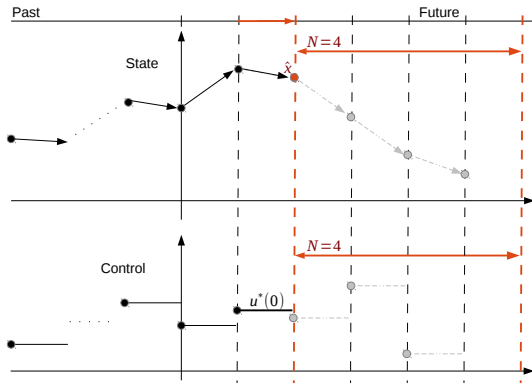
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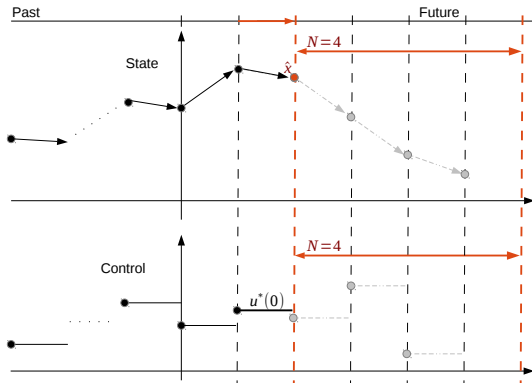


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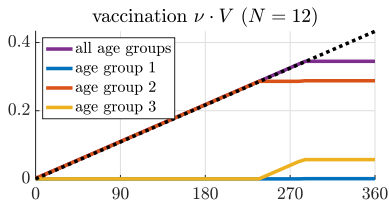
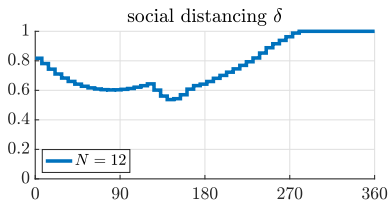
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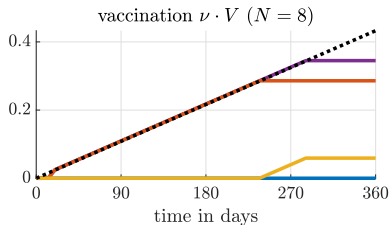
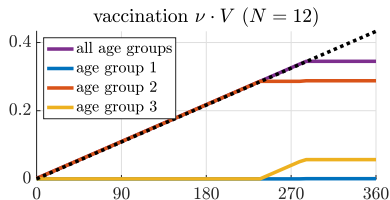
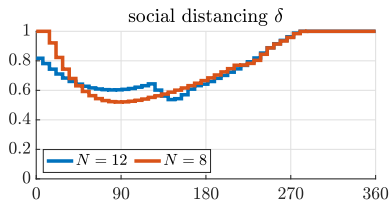
# Closed-loop simulations

## Impact of prediction horizon $N$ (in weeks) on controls



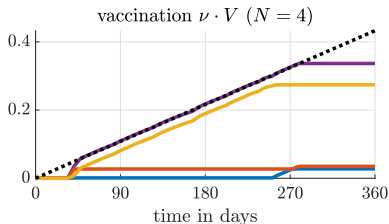
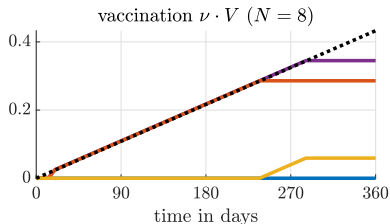
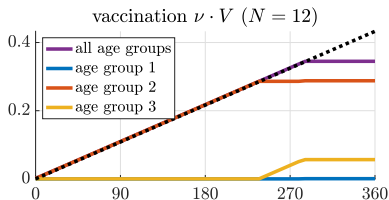
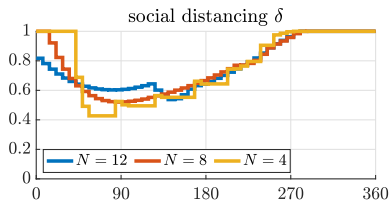
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# Conclusions & outlook

## Recap

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- breakpoints/bifurcation

# References

## The presented results are based on

[1] S. Grundel, S. Heyder, T. Hotz, T. K. S. Ritschel, P. Sauerteig, K. Worthmann

**How much testing and social distancing is required to control COVID-19? Some insight based on an age-differentiated compartmental model**

Submitted. (Preprint available at <https://arxiv.org/abs/2011.01282>)

[2] S. Grundel, S. Heyder, T. Hotz, T. K. S. Ritschel, P. Sauerteig, K. Worthmann

**How to Coordinate Vaccination and Social Distancing to Mitigate SARS-CoV-2 Outbreaks**

Submitted. (Preprint available at medRxiv, DOI: 10.1101/2020.12.22.20248707)

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## The parameters are taken from

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[6] X. He, E. H. Y. Lau, P. Wu, X. Deng, J. Wang, X. Hao, Y. C. Lau, J. Y. Wong, Y. Guan, X. Tan, X. Mo, Y. Chen, B. Liao, W. Chen, F. Hu, Q. Zhang, M. Zhong, Y. Wu, L. Zhao, F. Zhang, B. J. Cowling, F. Li, G. M. Leung

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**The incubation period of coronavirus disease 2019 (COVID-19) from publicly reported confirmed cases: estimation and application**

Ann. Med. 172(9), pp. 577–582



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## The parameters are taken from

[8] J. Mossong, N. Hens, M. Jit, P. Beutels, K. Auranen, R. Mikolajczyk, M. Massari, S. Salmaso, G. S. Tomba, J. Wallinga, J. Heijne, M. Sadkowska-Todys, M. Rosinska, W. J. Edmunds  
**Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases**  
PLoS Medicine, 5 (2008), pp. 381–391

[9] M. Park, A. R. Cook, J. T. Lim, Y. Sun, B. L. Dickens  
**A systematic review of COVID-19 epidemiology based on current evidence**  
J. Clin. Med., 9 (2020), p. 967

[10] J. Schilling, M. Diercke, D. Altmann, W. Haas, S. Buda  
**Vorläufige Bewertung der Krankheitsschwere von COVID-19 in Deutschland basierend auf übermittelten Fällen gemäß Infektionsschutzgesetz**

[11] R. Woelfel, V. M. Corman, W. Guggemos, M. Seilmaier, S. Zange, M. A. Mueller, D. Niemeyer, P. Vollmar, C. Rothe, M. Hoelscher, T. Bleicker, S. Bruenink, J. Schneider, R. Ehmann, K. Zwirgmaier, C. Drosten, C. Wendtner  
**Clinical presentation and virological assessment of hospitalized cases of coronavirus disease 2019 in a travel-associated transmission cluster**

## Thank you for your attention!

# Parameters

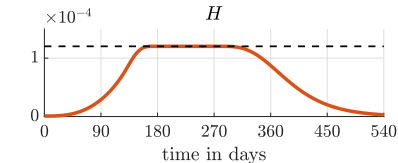
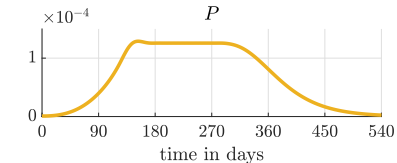
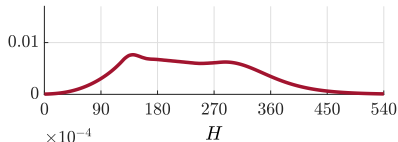
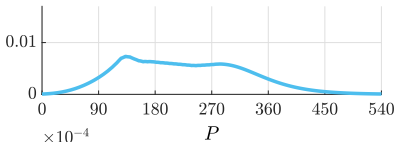
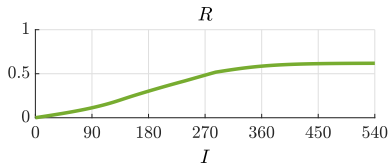
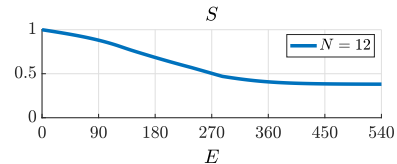
Description	Symbol	Value
Number of age groups	$n_g$	3
Regularization parameter	$\kappa$	$10^{-3}$
Removal rate (severe)	$\eta^S$	0.2500
Removal rate (mild)	$\eta^M$	0.2500
Removal rate (asymptomatic)	$\eta^A$	0.1667
Rate of becoming infectious	$\gamma$	0.1923
ICU admittance rate	$\rho$	0.0910
ICU discharge rate	$\sigma$	0.0952
Vaccine production limit	$V^{\max}$	100,000
Success rate	$q$	0.9

## Age-differentiated parameters

Age group	$i$	1	2	3
Age range (in years)	–	< 15	15 – 59	> 60
Relative age group size	$N_i$	0.1370	0.5776	0.2854
Probability of severe symptoms	$\pi_i^S$	0.0053	0.0031	0.0302
Probability of mild symptoms	$\pi_i^M$	0.1211	0.2201	0.2512
Probability of no symptoms	$\pi_i^A$	0.8737	0.7768	0.7186
Transmission rate (age group 1)	$\beta_{1i}$	0.4612		
Transmission rate (age group 2)	$\beta_{2i}$	0.4819	0.6304	
Transmission rate (age group 3)	$\beta_{3i}$	0.1243	0.2944	0.1802

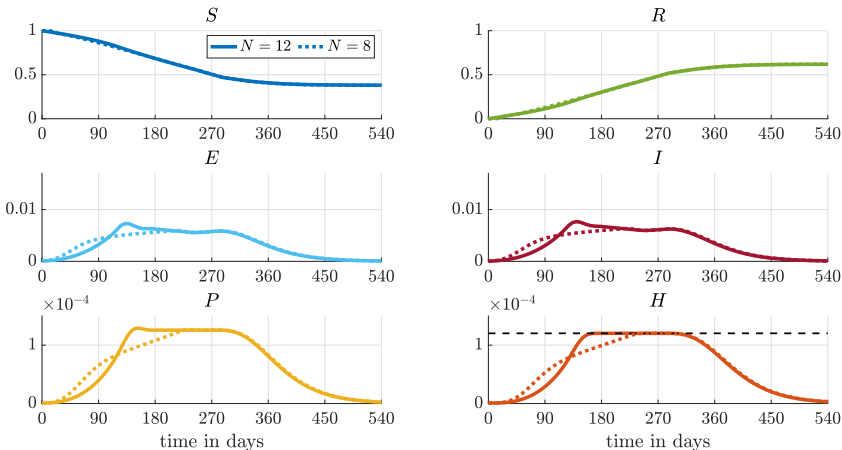
# Closed-loop simulations II

## Impact of prediction horizon $N$ (in weeks) on states



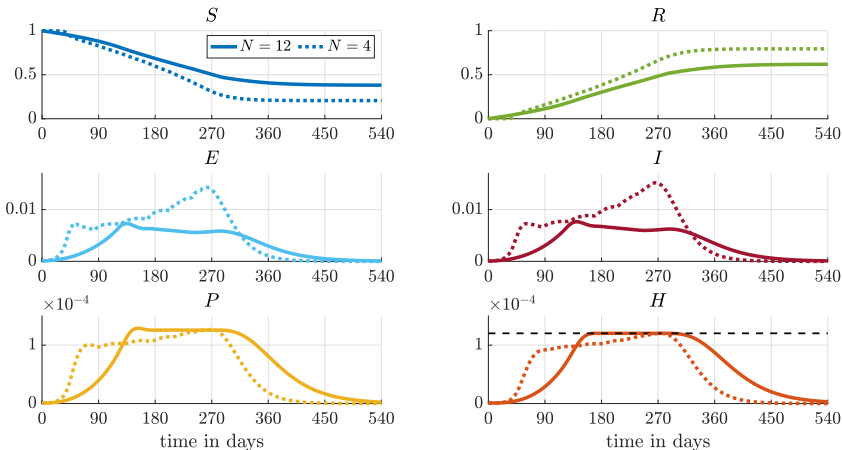
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# Minimizing fatalities

**Assumption:** total number of fatalities  $\propto$  total number of people treated on ICU

For given  $\delta^c \in [0, 1]$  solve

$$\min_{\nu} H^C(t_f) + \kappa \|\nu\|_2^2$$

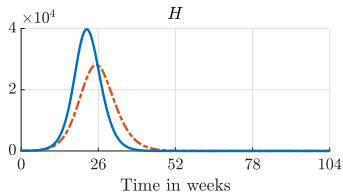
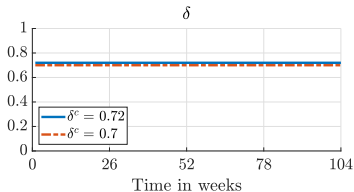
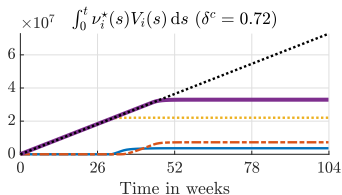
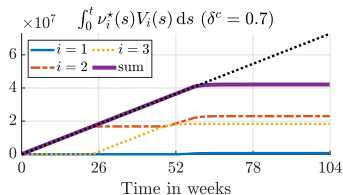
$$\text{subject to } \dot{H}^C(t) = \sum_{i=1}^{n_g} \sigma(H_i(t) + H_i^V(t)), \quad H^C(0) = 0$$

$$\dot{x}(t) = f(x(t), \delta^c, \nu(t)), \quad x(0) = x^0$$

$$\int_0^t \sum_{i=1}^{n_g} \nu_i(s) V_i(s) ds \leq V^{\max} \cdot t \quad \forall t \geq 0$$

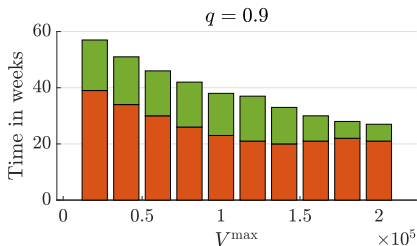
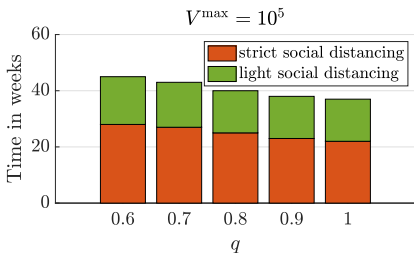
$$\nu(t) = \nu(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, \dots, N-1$$

# Minimizing fatalities



- contact restrictions sufficiently strict  $\rightsquigarrow$  vaccinate group with most contacts first
- otherwise  $\rightsquigarrow$  vaccinate high-risk group first ("damage control")

# Impact of $V^{\max}$ and $q$



for convenience:

- light contact restrictions:  $0.8 \leq \delta$
- strict contact restrictions:  $0.6 \leq \delta < 0.8$
- lockdown:  $\delta < 0.6$