MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

COMPUTATIONAL METHODS IN SYSTEMS AND CONTROL THEORY

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Surrogate Models for Coupled Microgrids

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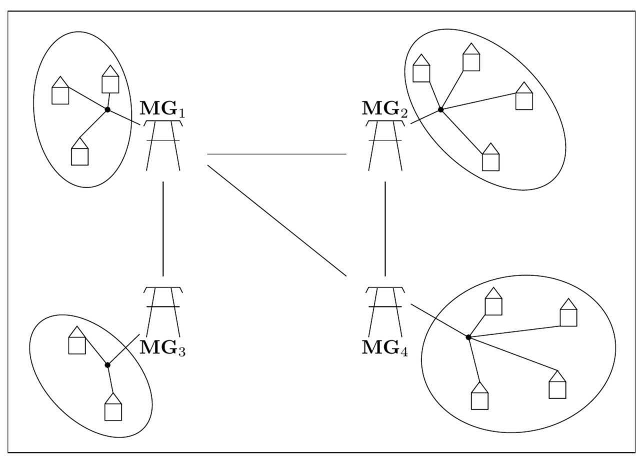
Coupled microgrids

We consider a network of coupled microgrids (MGs) where each MG consists of multiple *smart homes* equipped with photovoltaics and a personal battery. In a model predictive control (MPC) framework, the household batteries are used to compensate for the typical mismatch of power demand and generation in a 24-hours time interval at a single home.

When the MGs are *coupled*, further exchange of electricity is possible. This exchange among microgrids is beneficial in the German energy grid where power is typically produced in the North and demanded in the South.



The bilevel optimization problem

The control goal is to minimize the deviation from the overall average net consumption,

$$\bar{\zeta}_\kappa(n) := \frac{1}{\min\{N, n+1\}} \sum_{j=n-\min\{N-1,n\}}^{n} \bar{w}_\kappa(j), \quad \text{where } \bar{w}_\kappa(j) := \frac{1}{\mathcal{I}_\kappa} \sum_{i=1}^{\mathcal{I}_\kappa} w_i(j),$$

for the κ-th MG of size $\mathcal{I}_\kappa$. Here, N denotes the prediction horizon used in the following MPC problem:

Optimization among MGs: Consider a network of $\Xi \in \mathbb{N}$ coupled MGs, and denote by $\delta_{\kappa,\nu}$ the energy exchange rate from MG κ to MG ν. The following constrained optimization problem,

$$\mathbb{R}^{\Xi \times \Xi \times N} \ni \delta^* = \arg\min_{\delta} \sum_{n=k}^{k+N-1} \sum_{\kappa=1}^{\Xi} \left(\mathcal{I}_\kappa \bar{\zeta}_\kappa(n) - \sum_{\nu=1}^{\Xi} \delta_{\nu,\kappa}(n) \mathcal{I}_\nu \bar{z}_\nu(n) \right)^2 \tag{2a}$$

$$\text{subject to} \quad \delta_{\kappa,\nu}(n) \geq 0, \quad \sum_{\nu=1}^{\Xi} \delta_{\kappa,\nu}(n) = 1 \ \forall \kappa, \quad \text{and} \quad \delta_{\kappa,\nu}(n) \cdot \delta_{\nu,\kappa}(n) = 0 \text{ for all } \kappa \neq \nu, \tag{2b}$$

is solved using MatLab's built-in sequential quadratic programming (SQP) solver.

Optimization within a single MG: For a fixed prediction horizon N, obtain a forecast for $w_i(n)$ from the data set [4]. On this time horizon, an MPC problem for a single MG is formulated:

$$\mathbb{R}^{\mathcal{I} \times N} \ni u^* = \arg\min_{u} \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\bar{\zeta}(n) - \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) \right)^2, \quad k = 0, 1, 2, \ldots, \tag{3a}$$

subject to the *battery dynamics* in every RES written in time-discrete form,

$$x_i(n+1) = \alpha_i x_i(n) + \Delta t \left(\beta_i u_i^+(n) + u_i^-(n) \right), \tag{3b}$$

$$z_i(n) = w_i(n) + u_i^+(n) + \gamma_i u_i^-(n), \qquad n = k, \ldots, k+N-1, \tag{3c}$$

and additional box constraints on the state-of-charge x_i and on the charging and discharging rates u_i^+, u_i^-, for each $i = 1, \ldots, \mathcal{I}$.

The optimization problem (3a)-(3c) is solved using the alternating direction method of multipliers (ADMM) following [1].

Surrogate models in bilevel model predictive control

Main idea:

Replace the optimization problem (3a) – (3c) *via a surrogate model that maps,*

$$\left\{ x_1(k), \ldots, x_{\mathcal{I}}(k), \bar{w}(k), \ldots, \bar{w}(k+N-1), \bar{\zeta}(k), \ldots, \bar{\zeta}(k+N-1) \right\}_\nu \overset{\varphi_\nu}{\rightsquigarrow} \left\{ \bar{z}_\nu(k), \ldots, \bar{z}_\nu(k+N-1) \right\},$$

without computing the corresponding optimal control.

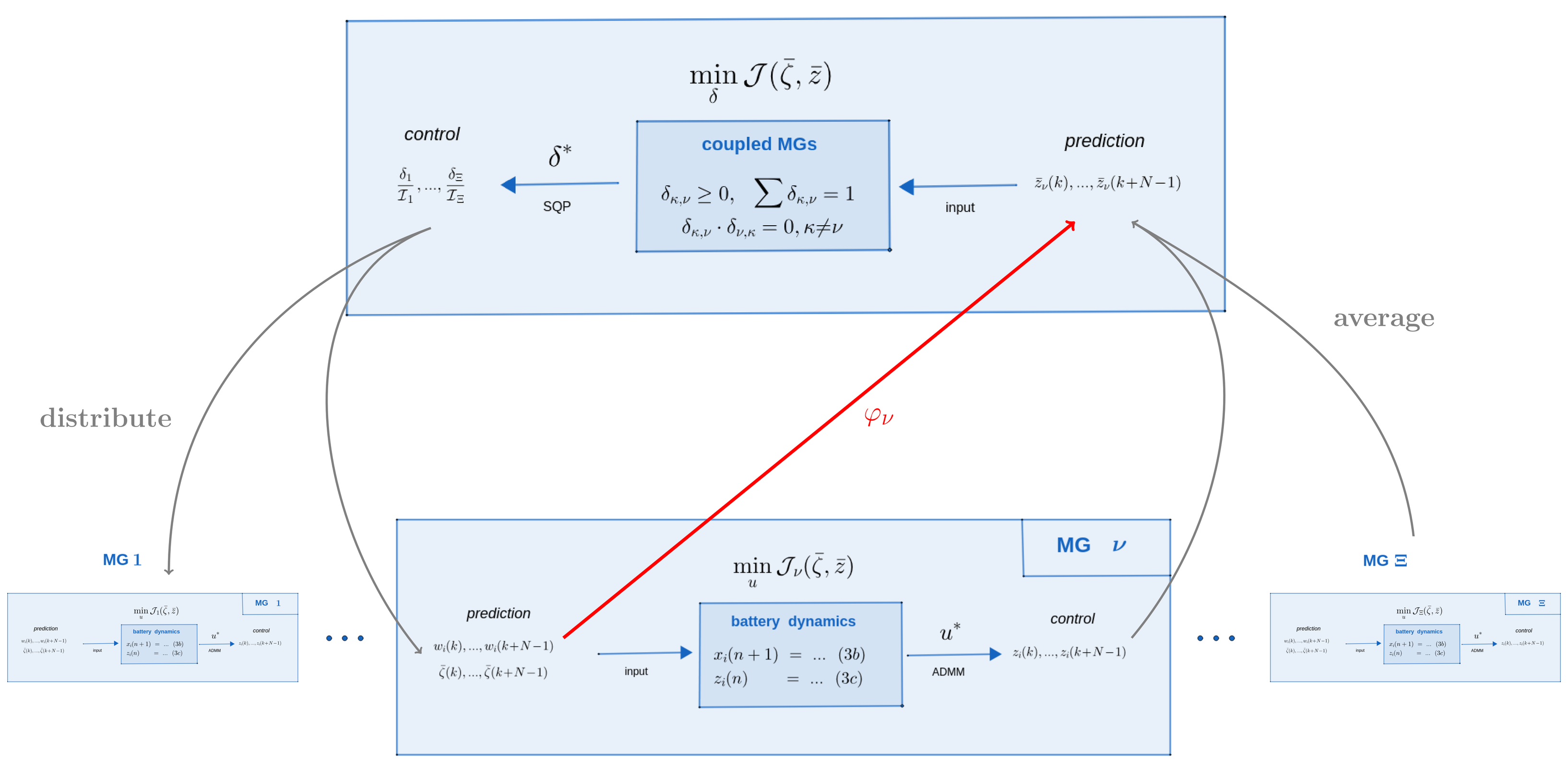
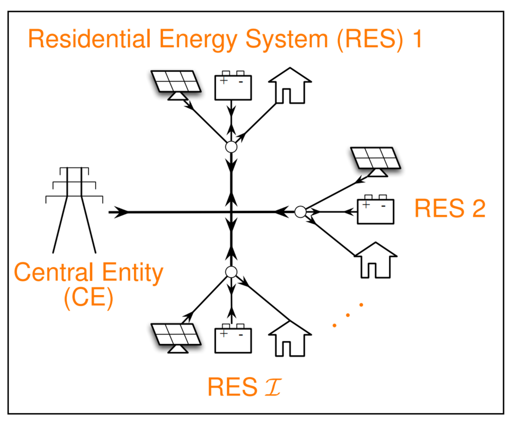


Figure 1: Bilevel optimization scheme with surrogate φ_ν replacing MG ν.

Residential energy systems



Each microgrid in (3b)-(3c) consists of $\mathcal{I} \in \mathbb{N}$ residential energy systems (RES) with:

- the battery's state-of-charge $x_i(k) \geq 0$,
- the battery's (dis-)charging rates $u_i^+(k) \geq 0$ and $u_i^-(k) \leq 0$, respectively,
- households with loads ℓ_i (power consumptions) and generations g_i (photovoltaics) resulting in the power demand in (3c), $w_i(k) := \ell_i(k) - g_i(k)$.

It is assumed that a central entity (CE) controls the batteries. The data is provided by the Austrailian grid company Ausgrid [4].

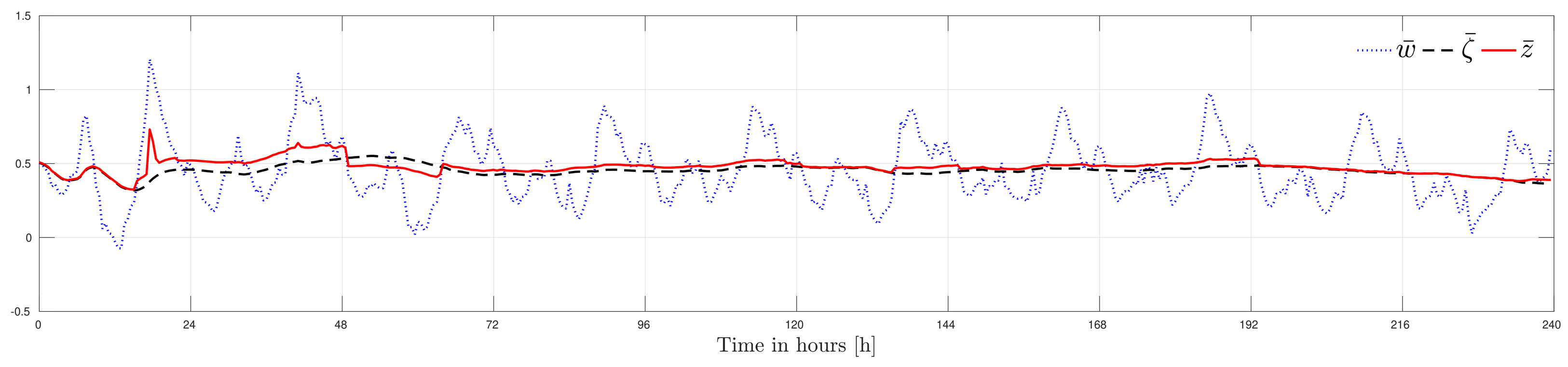


Figure 2: Uncontrolled aggregated power demand $\bar{w}(k) := \mathcal{I}^{-1} \sum_{i=1}^{\mathcal{I}} w_i(k)$ for $\mathcal{I} = 50$ households in Australia [4] measured every $\Delta t = 0.5h$. The optimization (3a)-(3c) yields a demand $\bar{z}(k)$ with much less fluctuations.

Radial basis functions vs. neural networks

We replace the mapping φ_ν by an approximation.

Radial basis functions: During an offline phase with M interpolation points, the approximant

$$\mathbb{R}^N \ni \varphi_{\mathrm{RB}}(\chi) = \sum_{i=1}^{M} \alpha_i \phi_i(\chi) + q(\chi), \quad \text{for } \chi \in \mathbb{R}^{2N+\mathcal{I}},$$

with basis functions $\phi_i(\chi) := \phi(\|\chi - \chi_i\|)$ is obtained using the MatLab toolbox DACE [3].

Neural network interpolation: In a learning phase, a four-layer neural network of the form,

$$\mathbb{R}^N \ni \varphi_{\mathrm{NN}}(\chi) = \sigma\left(W^{[4]} \sigma\left(W^{[3]} \sigma(W^{[2]} \chi + b^{[2]}) + b^{[3]} \right) + b^{[4]} \right), \quad \text{for } \chi \in \mathbb{R}^{2N+\mathcal{I}},$$

with σ being the sigmoid function, and two hidden layers of 10 neurons each is trained using MatLab's `nntraintool` function. The weights $W^{[i]}$ and biases $b^{[i]}$ are optimized based on a least-squares fit using M data points.

Numerical experiments:

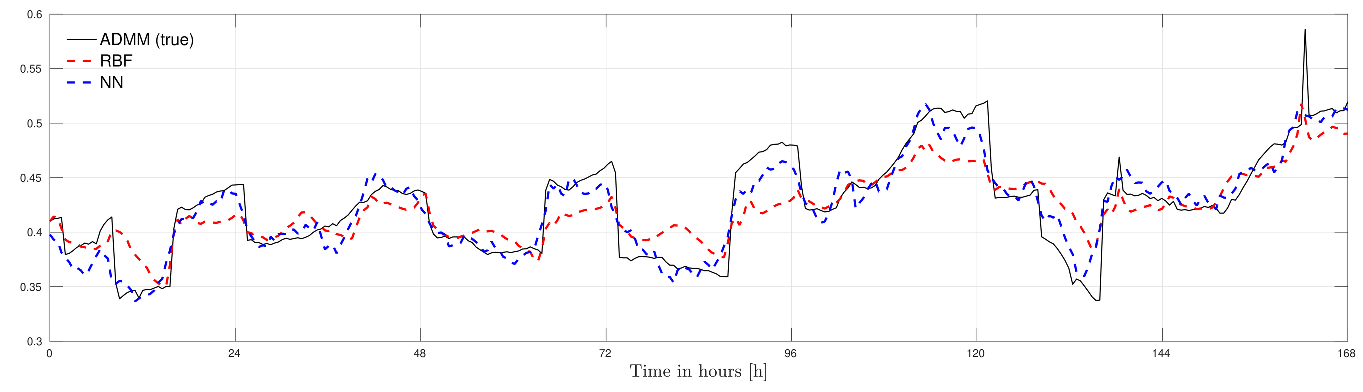


Figure 3: 7-days prediction of $\bar{z}(k)$ for a single MG after an offline/training phase with data from the preceding two weeks.

Conclusions and future research

Objectives of the ongoing research:

- Integration of a surrogate model using radial basis functions or neural networks as a partial replacement of ADMM within the bilevel optimization framework.
- Continuation of the work in [2] and, in particular, an extension to the entire MPC optimization loop. Analysis of necessary *updates* of the surrogate model during the MPC cycle.
- Extension of the mathematical model to line losses.

References

Ausgrid data set: The half-hour electricity data is for 300 homes with rooftop solar systems that are measured by a gross meter that records the total amount of solar power generated every 30 minutes. The data has been sourced from randomly selected customers in Ausgrid's electricity network area that were billed on a domestic tariff and had a gross metered solar system installed.

Related publications:

[1] P. Braun, T. Faulwasser, L. Grüne, C. M. Kellett, S. R. Weller, and K. Worthmann, *Hierarchical distributed ADMM for predictive control with applications in power networks*, IFAC Journal of Systems and Control, 3 (2018), pp. 10–22.

[2] S. Grundel, P. Sauerteig, and K. Worthmann, *Surrogate models for coupled microgrids*, in Progress in Industrial Mathematics at ECMI 2018, Springer, 2018 (submitted).

[3] H. B. Nielsen, S. N. Lophaven, and J. Søndergaard, *DACE - A Matlab Kriging Toolbox*, 2002.

[4] E. L. Ratnama, S. R. Wellera, C. M. Kelletta, and A. T. Murray, *Residential load and rooftop PV generation: an Australian distribution network dataset*, International Journal of Sustainable Energy, 36 (2017), pp. 787–806.