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Control of Residential Energy Systems using Energy Storages & Controllable Loads

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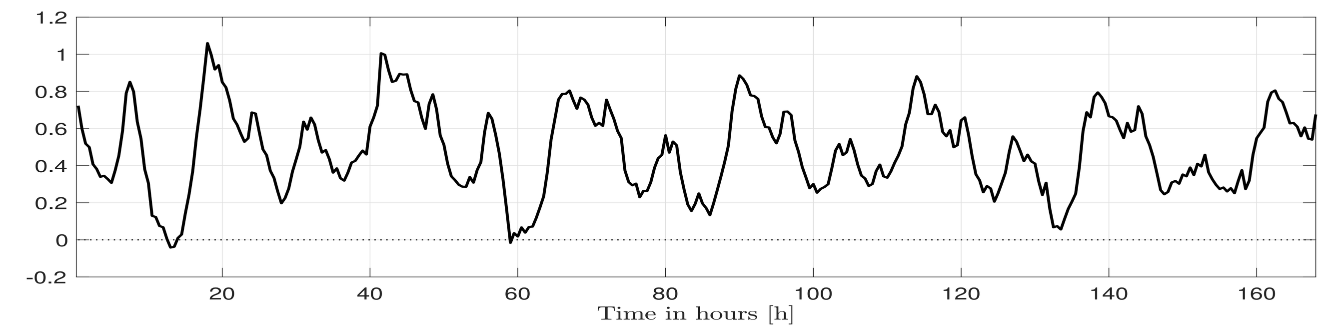
KONSENS

Project: Consistent Optimization and Stabilization of Electrical Networked Systems
Subproject: Distributed Optimization and Control of Microgrids

Present Situation

Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} w_i$



Data provided by an Australian electricity network.
Problem: Fluctuations of the power demand
Idea: Exploit flexibilities: storage devices

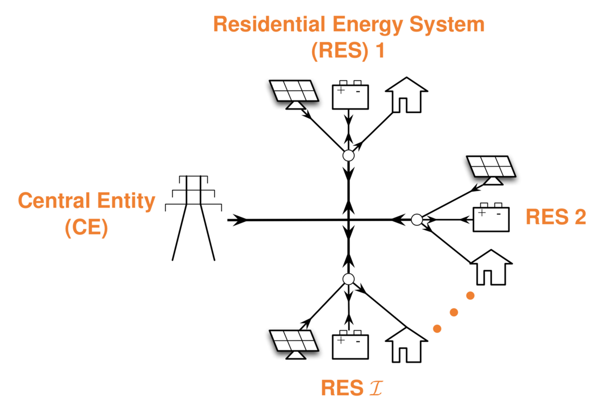
Residential Energy Systems (RESs) [4], [5]

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)
System equation of RES $i \in [1:\mathcal{I}] = \{1,\dots,\mathcal{I}\}$ at time $k \in \mathbb{N}_0$:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \end{aligned}$$

Constraints: For all $i \in [1:\mathcal{I}]$ and all $k \in \mathbb{N}_0$

$$\begin{aligned} 0 \le{} & x_i(k) && \le C_i \\ \underline{u}_i \le{} & u_i^-(k) && \le 0 \\ 0 \le{} & u_i^+(k) && \le \overline{u}_i \\ 0 \le{} & \tfrac{u_i^-(k)}{\underline{u}_i} + \tfrac{u_i^+(k)}{\overline{u}_i} && \le 1 \end{aligned}$$



Notation

- State of charge $x_i(k) \ge 0$ of the battery
- Power demand $z_i(k) \in \mathbb{R}$
- Net consumption $w_i(k) = \ell_i(k) - g_i(k) \in \mathbb{R}$ (load minus generation)
- Charging rate $u_i^+(k) \ge 0$ and discharging rate $u_i^-(k) \le 0$
- Sampling interval length $T > 0$
- Losses $\alpha_i, \beta_i, \gamma_i \in (0,1]$ due to energy transformation

Problem Formulation

Objective: Minimize the deviation from the overall average net consumption

$$\overline{\zeta}(k) = \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \zeta_i(k),$$

where

$$\zeta_i(k) = \begin{cases} \frac{1}{k+1}\sum_{n=0}^{k} w_i(n) & \text{if } k < N-1, \\ \frac{1}{N}\sum_{n=k-N+1}^{k} w_i(n) & \text{if } k \ge N-1. \end{cases}$$

Opimization Problem (finite time horizon $N \in \mathbb{N}_{\ge 2}$)

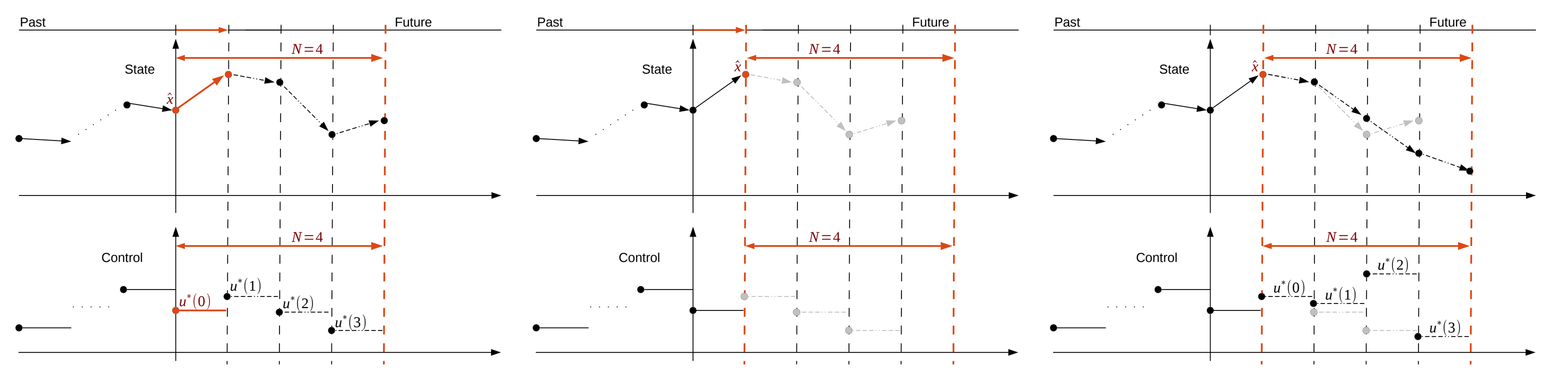
$$\min_{\mathbf{u}=(\mathbf{u}^+,\mathbf{u}^-)} \frac{1}{N}\sum_{n=k}^{k+N-1}\left(\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n)\right)^2 \qquad \text{(OCP)}$$
$$\text{s.t. system dynamics and constraints}$$

ADMM Formulation

$$\min_{\mathbf{u},\mathbf{a}} \frac{1}{N}\sum_{n=k}^{k+N-1}\left(\overline{\mathbf{a}}(n) - \overline{\zeta}(n)\right)^2 = \frac{1}{N}\|\overline{\mathbf{a}} - \overline{\zeta}\|_2^2$$
$$\text{s.t. system dynamics and constraints}$$
$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in [k:k+N-1]$$

Model Predictive Control (MPC)

Basic Idea:



Input: Time horizon $N \in \mathbb{N}$, number of systems $\mathcal{I} \in \mathbb{N}$, net consumption $w_i(n)$, $i \in [1:\mathcal{I}]$, $n \in [k:k+N-1]$.
Initialization: Set $k = 0$.
Main loop: For $k \in \mathbb{N}_0$

(1) For all $i \in [1:\mathcal{I}]$, measure the current states $\hat{x}_i := x_i(k)$.
(2) Solve (OCP) to obtain minimizing sequences

$$\mathbf{u}_i^\star = (u_i^\star(k),\dots,u_i^\star(k+N-1))^T \qquad \forall\, i \in [1:\mathcal{I}].$$

(3) For all $i \in [1:\mathcal{I}]$, implement $\mu_i(k,\hat{x}_i) := u_i^\star(k)$, shift the horizon forward in time, i.e. set $k = k+1$, and go to Step (1).

Alternating Direction Method of Multipliers (ADMM) [1]

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.
Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)
Loop: While $\ell \le \ell_{\max}$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2}\|\mathbf{z}_i - \mathbf{a}_i^\ell\|_2^2$$

for each RES $i \in [1:\mathcal{I}]$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

2. The CE solves

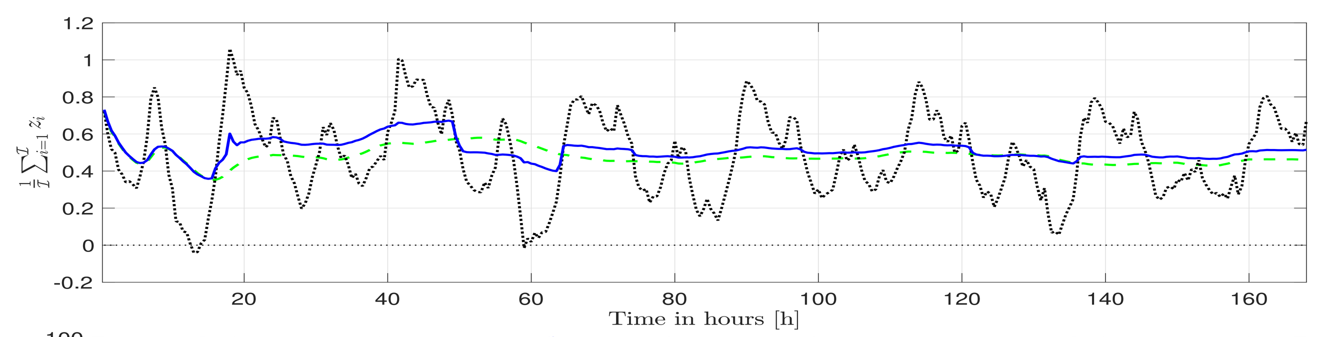
$$\mathbf{a}^{\ell+1} \in \arg\min_{\mathbf{a}} \|\overline{\mathbf{a}} - \overline{\zeta}\|_2^2 - \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i^\top \lambda_i^\ell + \frac{\rho}{2}\|\mathbf{z}_i^{\ell+1} - \mathbf{a}_i\|_2^2.$$

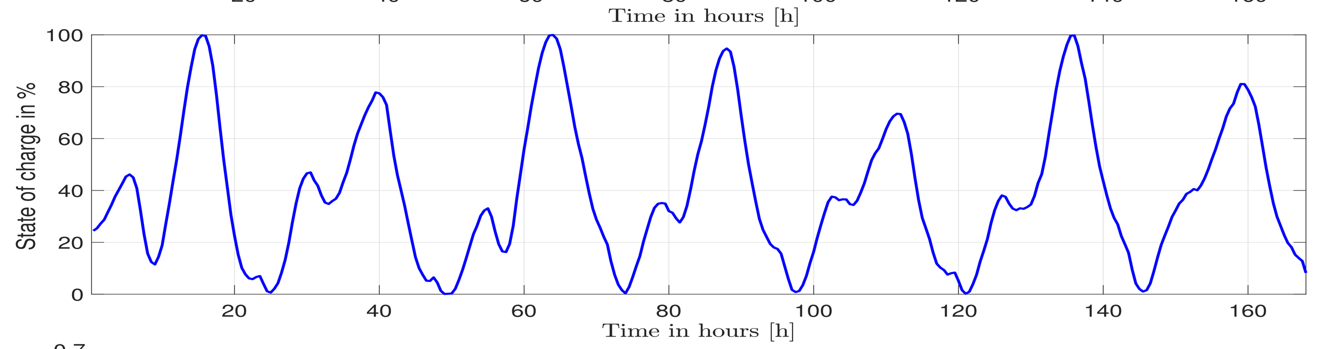
3. The CE updates the Lagrange multipliers

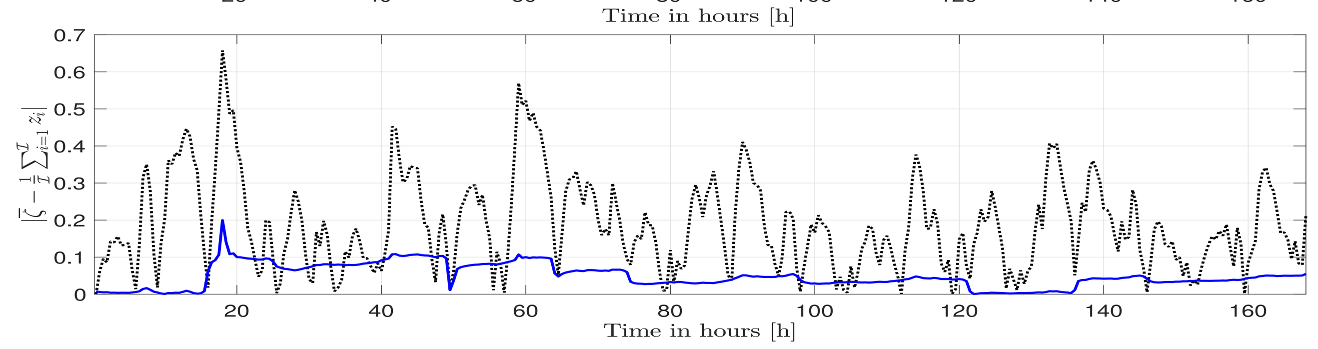
$$\lambda_i^{\ell+1} = \lambda_i^\ell + \rho(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i^{\ell+1}) \quad \forall\, i \in \{1,\dots,\mathcal{I}\}$$

and broadcasts $(\lambda_i^{\ell+1}, \mathbf{a}_i^{\ell+1})$ to RES $i \in [1:\mathcal{I}]$. Set $\ell = \ell + 1$.

Numerical Results







Observations

- Significant peak shaving of the overall performance
- Deviations from the desired reference value due to battery capacities and (dis)charging rates

Controllable Loads [2]

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

Additional Constraints

$$0 \le u_i^c(k) \le \overline{w}_i^c$$
$$\sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \le u_i^c(k) \le \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j)$$

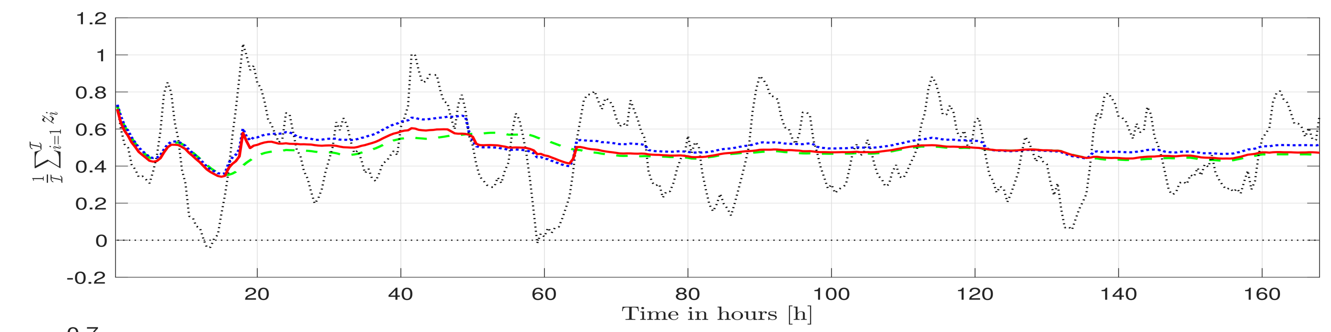
for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

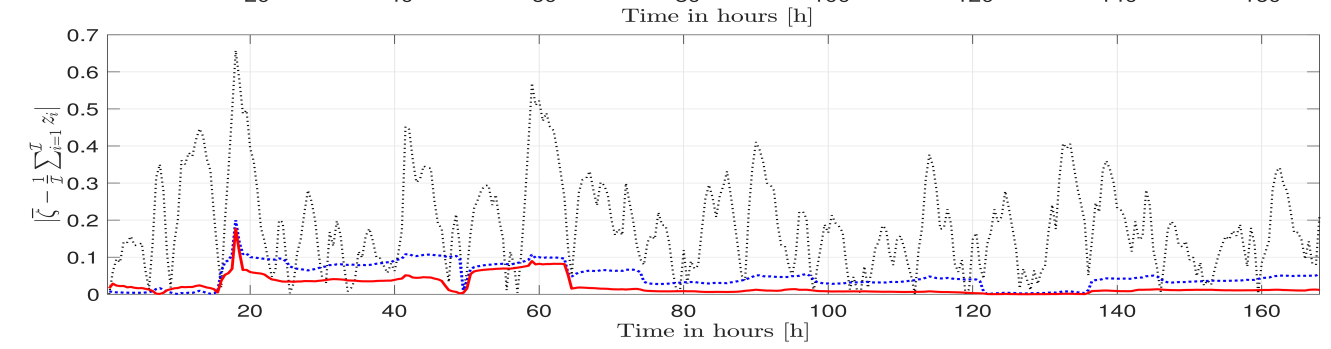
System Dynamics

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i^s(k) + u_i^+(k) + \gamma_i u_i^-(k) + u_i^c(k) \end{aligned}$$

$\rightsquigarrow$ Additional optimization variable u^c

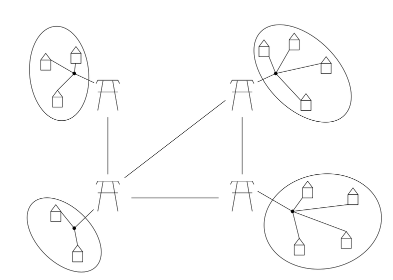
Observations
Further improvement of the overall performance





Outlook

- Coupled microgrids [4]



First numerical simulations show potential, but no convergence analysis so far.

- Surrogate models, e.g. to speed up the computation of single micro grids
- Uncertainities, e.g. time series analysis to identify outliers
- Multi-objective optimization: Peak shaving vs. tube constraints
- Price-based control [3]

References

[1] P. Braun, T. Faulwasser, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Hierarchical Distributed ADMM for Predictive Control with Applications in Power Networks, *IFAC Journal of Systems and Control*, **3**, 10-22, 2018

[2] P. Braun, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Model Predictive Control of Residential Energy Systems Using Energy Storage & Controllable Loads, *Progress in Industrial Mathematics at ECMI 2014. Mathematics in Industry*, **22**, 617-623, 2016

[3] P. Braun, L. Grüne, C. M. Kellett, S. R. Weller and K. Worthmann. Towards price-based predictive control of a small scale electricity network, *International Journal of Control*, 0(**0**), 1-22, 2017

[4] P. Braun, P. Sauerteig, K. Worthmann. Distributed optimization based control on the example of microgrids, *Computational Intelligence and Optimization Methods for Control Engineering*, Submitted, 2018

[5] K. Worthmann, C.M. Kellett, P. Braun, L. Grüne and S.R. Weller. Distributed and decentralized control of residential energy systems incorporating battery storage, *IEEE Trans. Smart Grid*, 6(**4**), 1914-1923, 2015