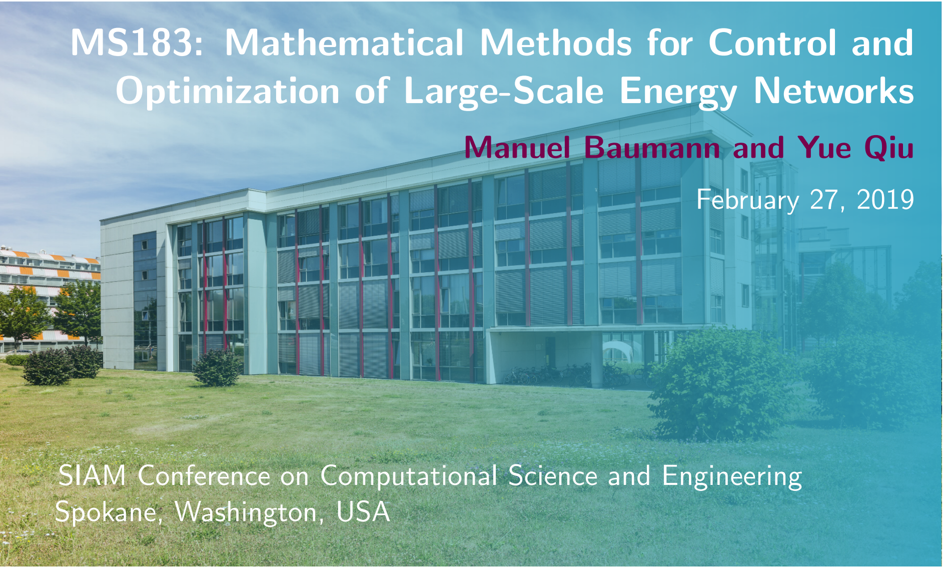
MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC

COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY



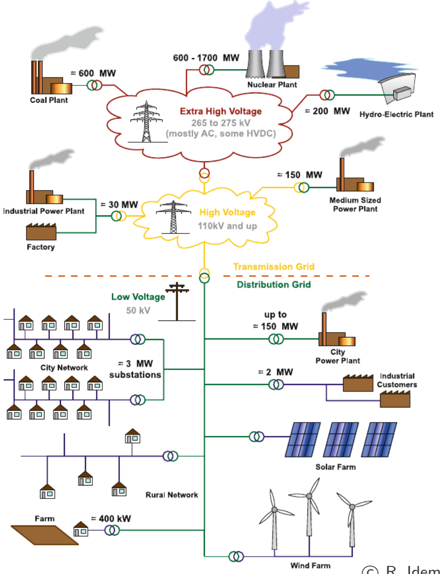
MS183: Mathematical Methods for Control and Optimization of Large-Scale Energy Networks

Manuel Baumann and Yue Qiu

February 27, 2019

SIAM Conference on Computational Science and Engineering
Spokane, Washington, USA

Aim of this mini-symposium



The energy network is subject to ongoing changes:

- renewables
- electric car
- batteries
- power-to-gas

Gives rise to new mathematical challenges!

Today's schedule

Part I/II

Manuel Baumann	Model-reduction for Dynamic Power Flow
Domenico Lahaye	Newton-Krylov Methods for the PFE
Riccardo Morandin	Hierarchical Modeling of Power Networks
Baljinnyam Sereeter	Four Mathematical Formulations of the OPF

Today's schedule

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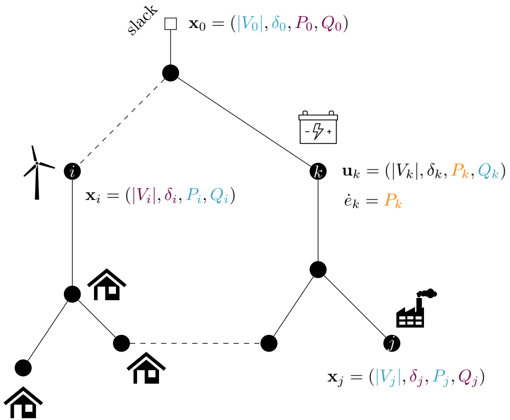
Part II/II at Room 302A (2:15 PM – 3:55 PM)

Peter Benner ~~Yue Qiu~~	Numerical Methods for Gas Networks
Stephan Gerster	Fluctuations in Supply Networks
Jennifer Uebbing	Optimization of Power-to-methane Processes
Anne Markensteijn	Load Flow Analysis of Multi-carrier Energy Systems

Model-order Reduction for Dynamic Power Flow Simulations

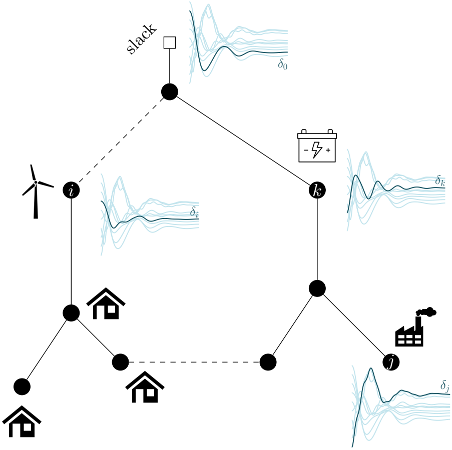
Static power flow simulation



Power flow equations

Nonlinear relation between the voltage $V_i = |V_i|e^{-j\delta_i}$ and the power $S_i = P_i + jQ_i$ at node i.

Dynamic power flow simulation



Power flow equations

Nonlinear relation between the voltage $V_i = |V_i|e^{-j\delta_i}$ and the power $S_i = P_i + jQ_i$ at node i.

Swing equations

ODE's on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$,

$$\ddot{\delta}_i + \dot{\delta}_i = P_i - \sum_{j \neq i} \sin(\delta_i - \delta_j)$$

[coefficients omitted.]

Dynamic power flow simulation

Three leading models

Governing equations (*swing equations*) at network node $i \in \mathcal{V}$,

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = A_i - \sum_{j \neq i} K_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}), \quad i = 1, ..., N,$$

yield for a specific choice of parameters the three models

EN **effective network model** ($N = |\mathcal{V}_{\text{gen}}|$),

SM **synchronous motor model** ($N = |\mathcal{V}|$),

SP structure-preserving model ($N > |\mathcal{V}|$).

T. Nishikawa and A. E. Motter (2015). *Comparative analysis of existing models for power-grid synchronization.* New Journal of Physics 17:1.

Linear MOR in a nutshell

A linear dynamical system,

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x}$$

is approximate by a reduced-order model,

$$\dot{\hat{\mathbf{x}}} = \hat{A}\hat{\mathbf{x}} + \hat{B}\mathbf{u}$$
$$\hat{\mathbf{y}} = \hat{C}\hat{\mathbf{x}}$$

such that the output difference $\|\mathbf{y} - \hat{\mathbf{y}}\|$ is small.

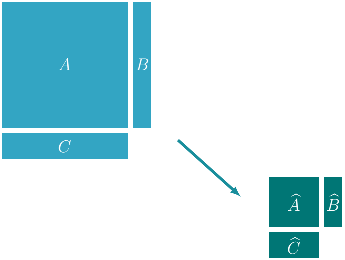


Figure: Petrov-Galerkin projections $\hat{A} := W_r^T A V_r$, $\hat{B} := W_r^T B$, and $\hat{C} := C V_r$.

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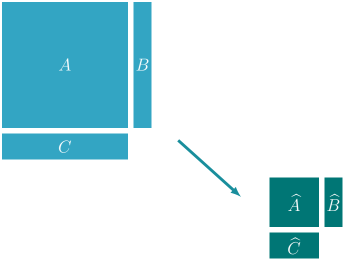


Figure: Petrov-Galerkin projections $\hat{A} := W_r^T A V_r$, $\hat{B} := W_r^T B$, and $\hat{C} := C V_r$.

Note: Projection of nonlinearity $W_r^T f(V_r \hat{x})$ *requires hyper-reduction.*

Swing equations are **non**linear,

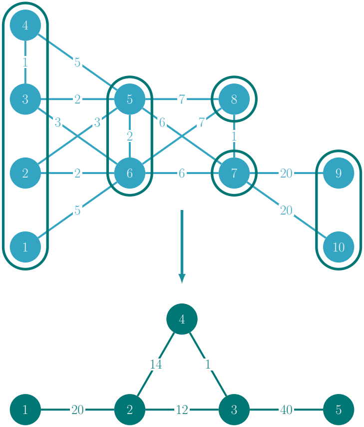
$$\dot{x} = f(x,t), \quad x := [\delta, \dot{\delta}].$$

Projection/reduction:

$$\dot{\hat{x}} = W_r^T f(V_r \hat{x}, t).$$

Nonlinear MOR:

- $f(V_r \hat{x})$ still large
- hyper-reduction
- clustering: $V_r = W_r = P(\pi)$



Algorithm:

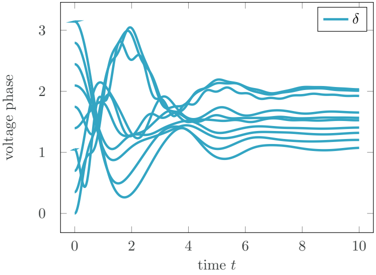
1. Collect snapshots

$$X := [\mathbf{x}(t_1), ..., \mathbf{x}(t_s)]$$

2. Principal components

$$X =: U\Sigma V^T \quad \leftarrow \text{SVD of } X$$

3. k-means clustering
4. Projection $P(\pi)$



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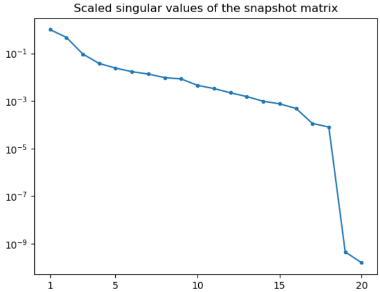

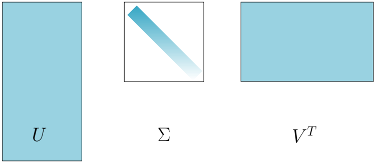

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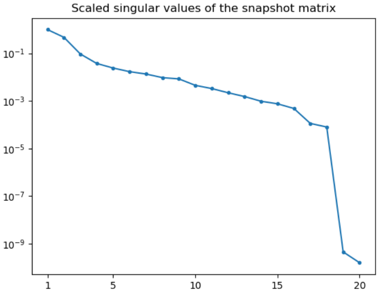

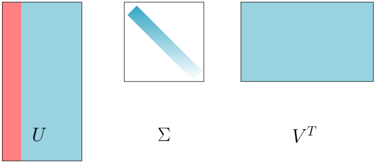

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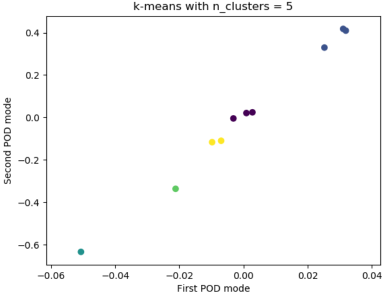

Model-Order Reduction on Graphs

Algorithm:

1. Collect snapshots

$$X := [\mathbf{x}(t_1), ..., \mathbf{x}(t_s)]$$

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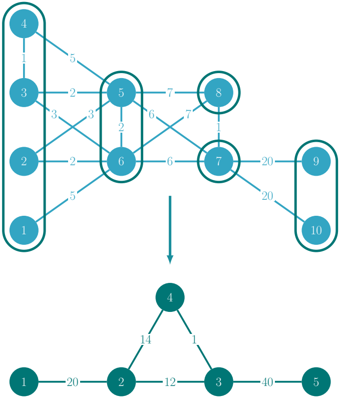
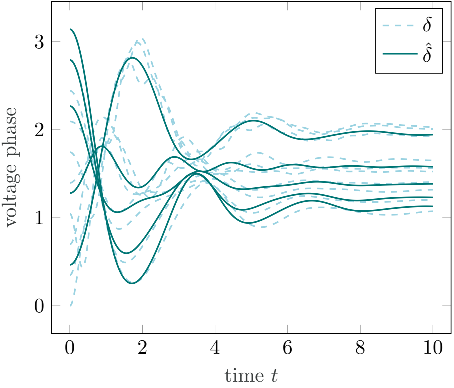
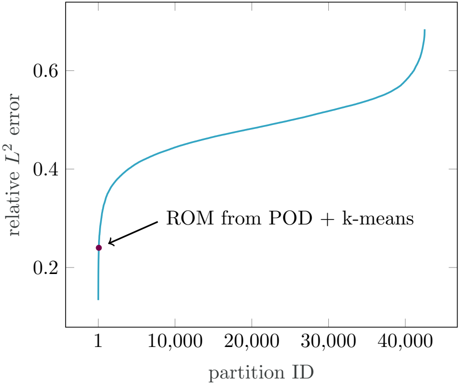


The approximation error:





The (simplified) swing equations,

$$H_i\ddot{\delta}_i + D_i\dot{\delta}_i = A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j), \quad i = 1, ..., N,$$

in vectorized form, yields a structure-preserving ROM:

$$H\ddot{\boldsymbol{\delta}} + D\dot{\boldsymbol{\delta}} = A - \left(K \odot \sin(\boldsymbol{\delta}\mathbf{1}_n{}^T - \mathbf{1}_n\boldsymbol{\delta}^T)\right)\mathbf{1}_n, \qquad \text{FOM}$$

$$\hat{H}\ddot{\hat{\boldsymbol{\delta}}} + \hat{D}\dot{\hat{\boldsymbol{\delta}}} = \hat{A} - \left(\hat{K} \odot \sin(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)\right)\mathbf{1}_r. \qquad \text{ROM}$$

Model-Order Reduction on Graphs

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The nonlinear term reduces to:

$$\begin{aligned} P^T\left(K \odot \sin(\boldsymbol{\delta}\mathbf{1}_n{}^T - \mathbf{1}_n\boldsymbol{\delta}^T)\right)\mathbf{1}_n &\approx P^T\left(K \odot \sin(P\hat{\boldsymbol{\delta}}(P\mathbf{1}_r)^T - P\mathbf{1}_r(P\hat{\boldsymbol{\delta}})^T)\right)P\mathbf{1}_r \\ &= P^T\left(K \odot \sin(P(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)P^T)\right)P\mathbf{1}_r \\ &= \left(P^TKP \odot \sin(\hat{\boldsymbol{\delta}}\mathbf{1}_r{}^T - \mathbf{1}_r\hat{\boldsymbol{\delta}}^T)\right)\mathbf{1}_r \end{aligned}$$

Balanced truncation for a quadratic re-formulation of the swing equations

Reformulation to quadratic system

Swing equations in first-order form, $\omega := \dot{\delta}$,

$$\begin{bmatrix} \dot{\delta}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} \omega_i \\ \frac{\omega_R}{2H_i}\left(A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j - \gamma_{ij}) - \frac{D_i}{\omega_R}\omega_i\right) \end{bmatrix},$$

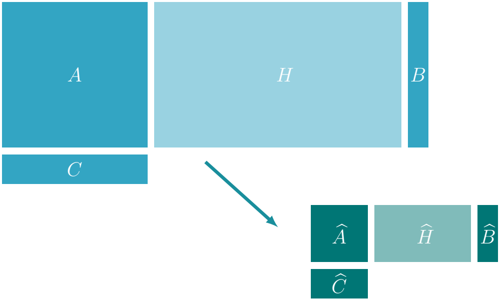
Quadratic formulation introducing $s_i := \sin(\delta_i)$ and $c_i := \cos(\delta_i)$,

$$\begin{bmatrix} \dot{\delta}_i \\ \dot{\omega}_i \\ \dot{s}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} \omega_i \\ \frac{\omega_R}{2H_i}\left(A_i - \sum_{j\neq i} K_{ij}(s_i c_j \gamma_{ij}^c - c_i s_j \gamma_{ij}^c - c_i c_j \gamma_{ij}^s - s_i s_j \gamma_{ij}^s) - \frac{D_i}{\omega_R}\omega_i\right) \\ c_i\omega_i \\ -s_i\omega_i \end{bmatrix},$$

with constants $\gamma_{ij}^c := \cos(\gamma_{ij})$ and $\gamma_{ij}^s := \sin(\gamma_{ij})$.

Input/output systems in quadratic form

$$\dot{x}(t) = Ax(t) + H\ x(t) \otimes x(t) + Bu(t),$$
$$y(t) = Cx(t), \quad x(0) = x_0.$$



Balanced truncation for quadratic systems

Consider a quadratic input/output system,

$$\dot{x}(t) = Ax(t) + H\ x(t) \otimes x(t) + Bu(t),$$
$$y(t) = Cx(t), \quad x(0) = x_0.$$

with $x_i := [\delta_i, \xi_i, s_i, c_i]$ and represented by matrices $\{A, B, C, H\}$.

Projection-based model-order reduction

Requires solution of two quadratic matrix equations

- $A\mathbf{P} + \mathbf{P}A^T + H(\mathbf{P} \otimes \mathbf{P})H^T = -BB^T, \qquad \mathbf{P} =: RR^T,$
- $A^T\mathbf{Q} + \mathbf{Q}A + H^{(2)}(\mathbf{P} \otimes \mathbf{Q})(H^{(2)})^T = -C^TC, \quad \mathbf{Q} =: SS^T.$

Obtain projection spaces based on truncated SVD of S^TR.

P. Benner, P. Goyal (2017). *Balanced Truncation Model Order Reduction For Quadratic-Bilinear Control Systems.* arXiv:1705.00160.

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Projection-based model-order reduction

Requires solution of two quadratic matrix equations

- $A_s\mathbf{P} + \mathbf{P}{A_s}^T + H(\mathbf{P} \otimes \mathbf{P})H^T = -BB^T, \qquad \mathbf{P} \approx RR^T,$
- ${A_s}^T\mathbf{Q} + \mathbf{Q}A_s + H^{(2)}(\mathbf{P} \otimes \mathbf{Q})(H^{(2)})^T = -C^TC, \quad \mathbf{Q} \approx SS^T.$

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Numerical experiments

Aim: Consider slack node dynamics as output, $y = Cx = x_1$, and isolate node x_1 when clustering.

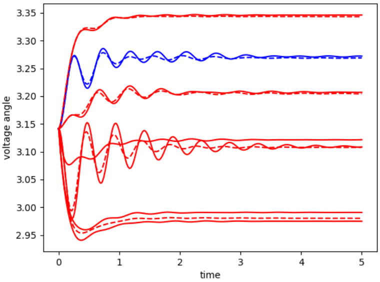


Figure: EN model clustering-based MOR.

57 nodes

7 generators

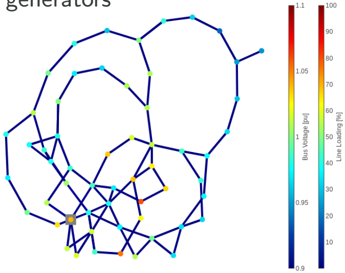
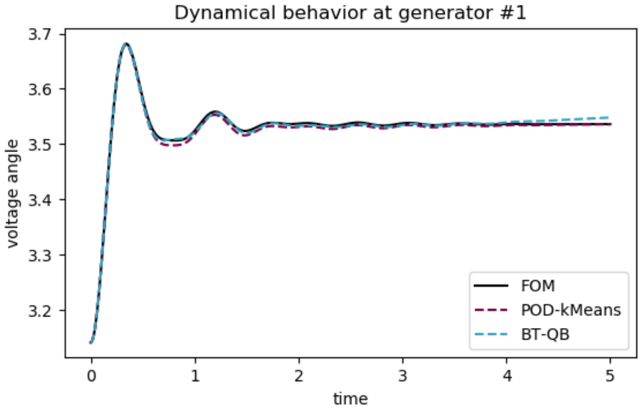


Figure: `IEEE Case57` taken from MATPOWER.

Numerical experiments



case57.m	FOM dim.	ROM dim.	rel. L_∞ error
EN model	14 vs. 28	10 vs. 19	0.0030 vs. 0.0016
SM model	114 vs. 228	36 vs. 175	0.0024 vs. 0.0032

Summary

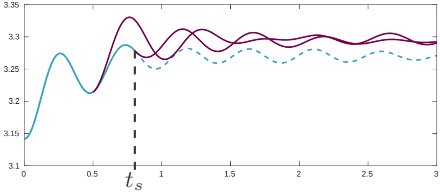
Conclusions:

- Structure-preserving ✓
- Stability-preserving **?**
- state-lifting in QB case ✗

Future work:

- Third-order swing equation, i.e. $|V_i| \rightsquigarrow |V_i(t)|$
- Fault recovery: line failure at time $t = t_s$ yields a switched system in quadratic form, i.e.

$$\{A_1, B_1, C_1, H_1\} \quad \overset{t_s}{\rightsquigarrow} \quad \{A_2, B_2, C_2, H_2\}.$$



Selected references

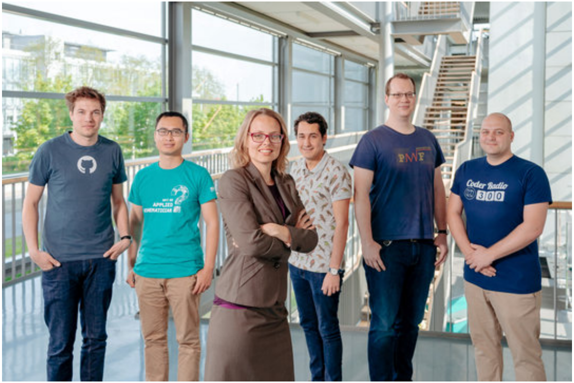
P. Benner and P. Goyal.
Balanced truncation model order reduction for quadratic-bilinear systems.
e-prints 1705.00160, arXiv, 2017.

P. Mlinarić, T. Ishizaki, A. Chakrabortty, S. Grundel, P. Benner, and J.-i. Imura.
Synchronization and aggregation of nonlinear power systems with consideration of bus network structures.
In *Proc. European Control Conf. (ECC)*, 2018.

F. Weiß.
Simulation, analysis, and model-order reduction for dynamic power network models.
Master's thesis, Otto-von-Guericke University Magdeburg, 2019 (ongoing).

The SES team at MPI Magdeburg

Thank you for your attention.
Question?



TenneT Taking power further

enso NETZ

VENIOS Uniting IT and Energy

energy saxony

http://konsens.github.io

Announcement

4th Workshop on Model Reduction of Complex Dynamical Systems - MODRED 2019 -

August 28th to 30th, 2019 in Graz

- Overview
- News
- Photos
- Scientific Program
- Participants
- Proceedings
- Location
- Social Activities
- Deadlines
- Abstracts
- Registration
- Accommodation
- Travel
- Useful links
- Contact

The conference starts Wednesday morning and ends on Friday. There will be **plenary talks** by a number of invited speakers. Moreover, there will be several **contributed talks** (20 minutes plus 5 minutes for questions and discussion).

Plenary Speakers

- Serkan Gugercin
- Bernard Haasdonk
- Dirk Hartmann (Siemens)
- Laura Iapichino
- J. Nathan Kutz

Contributed Talks

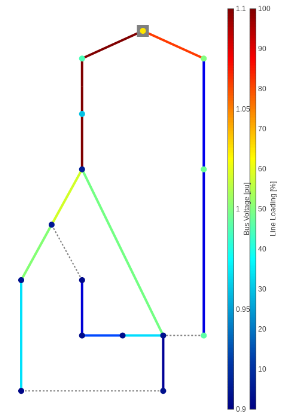
t.b.a.

Contact: modred2019@uni-graz.at

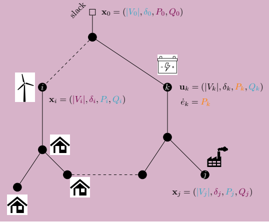
Last changed: 2018-06-19

Optimization of batteries

Test network



Mathematical model



At the nodes:

- voltage $V_i = |V_i| e^{j\delta_i}$
- power $S_i = P_i + jQ_i$
- batteries u_k

Optimization problem

$$\min_u \int_{t_0}^{t_e} \ell(V(u), t)\, \mathrm{d}t \qquad \text{(line loading)}$$

$$\text{s.t.} \quad c_E(V, S, t) + Bu(t) = 0 \qquad \text{(equal. constr.)}$$

$$\underline{x} \leq V_i(t), S_i(t) \leq \overline{x} \qquad \text{(state const.)}$$

$$\underline{u} \leq u_k(t) \leq \overline{u} \qquad \text{(control const.)}$$

Optimization of batteries

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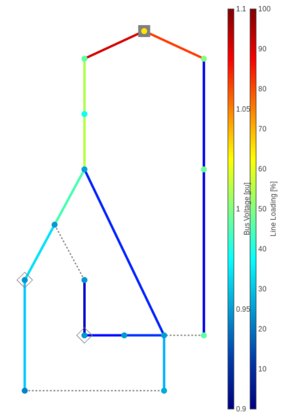

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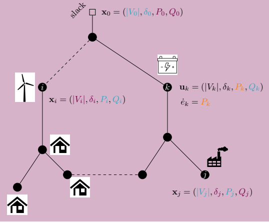

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