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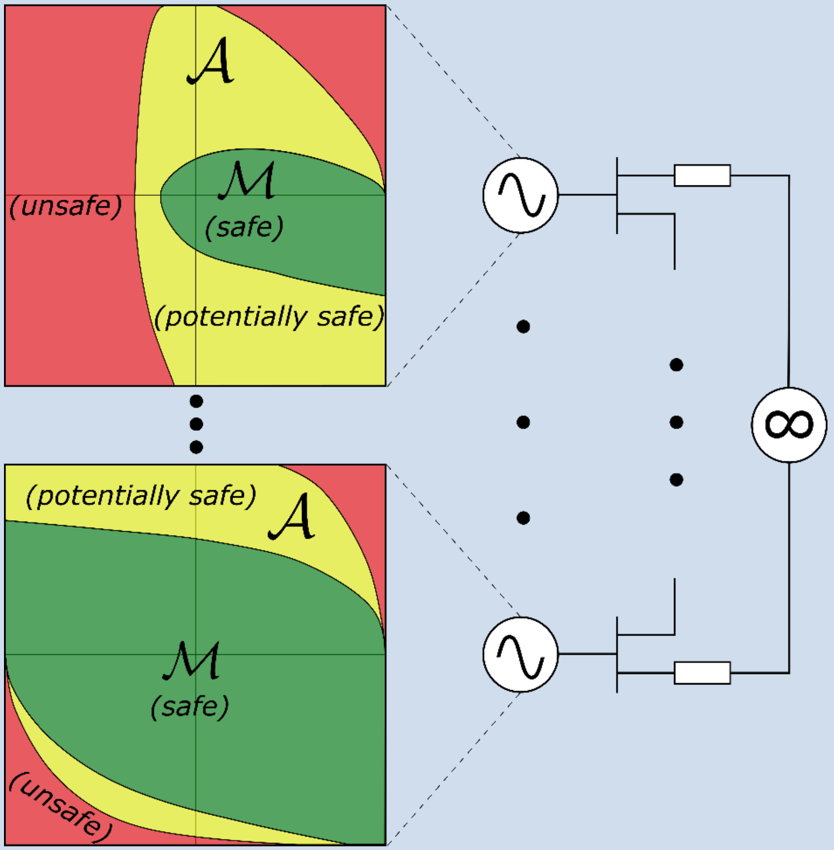
Transient Stability Analysis of Power Grids with Admissible & Maximal Robust Positively Invariant Sets



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Admissible sets and MRPIs: setting

Consider:

$$\dot{x}(t) = f(x(t), d(t)),$$
$$x(t_0) = x_0,$$
$$d \in \mathcal{D},$$
$$g_i(x(t)) \leq 0, \forall t \in [0, \infty[, \quad i = 1, 2, \dots, p.$$

Assumptions

(A1) The space $\mathcal{D}$ is the set of all Lebesgue measurable functions that map the interval $[t_0, \infty[$ to a set $D \subset \mathbb{R}^m$, which is compact and convex.

(A2) The function f is C^2 with respect to $d \in \mathcal{D}$, and for every d in an open subset containing D, the function f is C^2 with respect to $x \in \mathbb{R}^n$.

(A3) There exists a constant $0 < c < +\infty$ such that the following inequality holds true:

$$\sup_{d \in D} |x^T f(x, d)| \leq c(1 + \|x\|^2), \quad \text{for all } x \in \mathbb{R}^n.$$

(A4) The set $f(x, D) \triangleq \{f(x, d) : d \in D\}$ is convex for all $x \in \mathbb{R}^n$.

(A5) For every $i = 1, 2, \dots, p$, the function g_i is C^2 with respect to $x \in \mathbb{R}^n$.

➡ Most assumptions needed to have compactness of space of solutions.

Admissible sets and MRPIs

Definition: The *admissible set* (viability kernel) of the system is the set of initial states for which there exists an input $d \in \mathcal{D}$ such that the corresponding integral curve satisfies the constraints for all future time.

$$\mathcal{A} \triangleq \{x_0 \in \mathbb{R}^n : \exists d \in \mathcal{D},\ \ g_i(x^{(d,x_0,t_0)}(t)) \leq 0,\ \ \forall i,\ \ \forall t \in [\, t_0, \infty[\ \}.$$

Definition: A set $\Omega \subset \mathbb{R}^n$ is said to be a *robust positively invariant set* (RPI) of the system provided that $x^{(d,x_0,t_0)}(t) \in \Omega$ for all $t \in [\, t_0, \infty[$, for all $x_0 \in \Omega$ and for all $d \in \mathcal{D}$.

Let $G \triangleq \{x : g_i(x) \leq 0\ \forall i\}$

Definition: The *maximal robust positively invariant set* (MRPI) of the system contained in G, is the union of all RPIs that are subsets of G. Equivalently,

$$\mathcal{M} \triangleq \{x_0 \in \mathbb{R}^n : g_i(x^{(d,x_0,t_0)}(t)) \leq 0, \forall i, \forall d \in \mathcal{D},\ \ \forall t \in [\, t_0, \infty[\ \}.$$

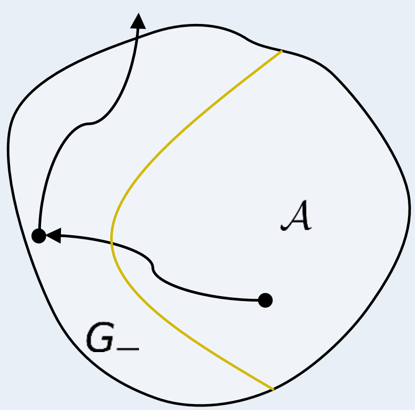
[De Dona and Levine: On barriers in state and input constrained nonlinear systems, SIAM Journal on Contr. & Opt., 2013]
[Esterhuizen, Aschenbruck, Streif: On Maximal Robust Positively Invariant Sets in Constrained Nonlinear Systems, under review]

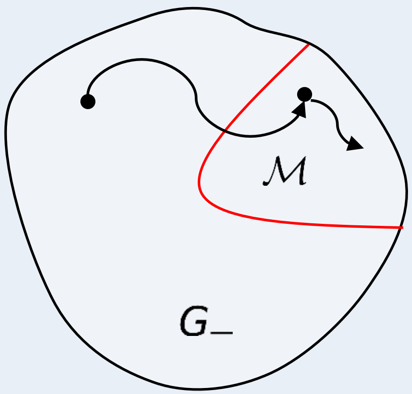
Proposition
Under assumptions (A1) – (A5) the sets $\mathcal{A}$ and $\mathcal{M}$ are closed.

➡ This follows from the compactness result

Let: $G_- \triangleq \{x : g_i(x) < 0 \forall i\}, \ [\partial \mathcal{A}]_- \triangleq \partial \mathcal{A} \cap G_-, \ [\partial \mathcal{M}]_- \triangleq \partial \mathcal{M} \cap G_-$

- These are special parts of the sets' boundaries called the *barriers*.
- Possess the *semi-permeability* property.





- Terms introduced by Isaacs in differential games.
- Barriers may be constructed via the Maximum Principle.

Stationarity

General

- Consider the set $\mathcal{M}_T \triangleq \{x_0 \in \mathbb{R}^n : x^{(d,x_0,t_0)}(t) \in G \; \forall t \in [t_0, T], \forall d \in \mathcal{D}\}$
- Clearly, $\mathcal{M} \subset \mathcal{M}_T$ and $\mathcal{M}_T$ is an RPI if and only if $\mathcal{M}_T \subset \mathcal{M}$

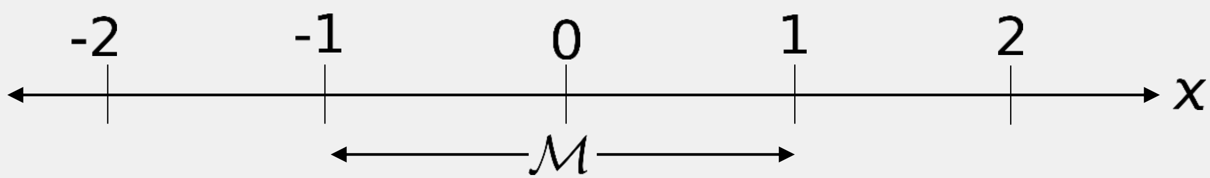
Definition

The set $\mathcal{M}$ is said to be stationary with horizon $T < \infty$ provided that:

$$\mathcal{M} = \mathcal{M}_T$$

Example

the set $\mathcal{M}$ for $\dot{x} = -x + d$, $|x| \leq 2$, $|d| \leq 1$ is not stationary.



Stationarity of $\mathcal{A}$

- Similar definition for the set $\mathcal{A}$.

➡ Stationarity guarantees that barriers intersect $G_0 \triangleq \partial G$ in finite time.

➡ We can now state our main theorem

Main Theorem

Theorem:
Consider a point $\bar{x} \in [\partial\mathcal{M}]_-$ (resp. $\bar{x} \in [\partial\mathcal{A}]_-$), assume that (A1)-(A5) hold and that the set $\mathcal{M}$ (resp. $\mathcal{A}$) is stationary with horizon $T < \infty$. Then there exists a $\bar{d} \in \mathcal{D}$ such that the corresponding integral curve runs along $[\partial\mathcal{M}]_-$ (resp. $[\partial\mathcal{A}]_-$) and intersects G_0 in finite time.
In other words, there exists a $\bar{d} \in \mathcal{D}$ and $\bar{t} \in [0, T]$ such that $x^{(\bar{d},\bar{x},t_0)}(t) \in [\partial\mathcal{M}]_-$ (resp. $x^{(\bar{d},\bar{x},t_0)}(t) \in [\partial\mathcal{A}]_-$) for all $t \in [t_0, \bar{t}[$, and $x^{(\bar{d},\bar{x},t_0)}(\bar{t}) \in G_0$. Moreover, this integral curve and the corresponding disturbance realization, $\bar{d} \in \mathcal{D}$, satisfy the following necessary conditions:

There exists a nonzero absolutely continuous maximal solution $\lambda^{\bar{d}}$ to the adjoint equation:

$$\dot{\lambda}^{\bar{d}}(t) = -\left(\frac{\partial f}{\partial x}(x^{(\bar{d})}(t), \bar{d}(t))\right)^T \lambda^{\bar{d}}(t),$$

$$\lambda^{\bar{d}}(\bar{t}) = (Dg_{i^*}(z))^T,$$

where, $z \triangleq x^{(\bar{d},\bar{x},t_0)}(\bar{t}) \in G_0$, such that…

Main Theorem (continued…)

$$\max_{d\in D}\{\lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), d)\} = \lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), \bar{d}(t)) = 0$$

$$\left(\text{resp.} \quad \min_{d\in D}\{\lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), d)\} = \lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), \bar{d}(t)) = 0\right)$$

for almost all $t \in [t_0, \bar{t}]$. Moreover, at time $\bar{t}$ the *ultimate tangentiality* condition holds:

$$\max_{d\in D}\max_{i\in \mathbb{I}(z)} L_f g_i(z, d) = \max_{i\in \mathbb{I}(z)} L_f g_i(z, \bar{d}(\bar{t})) = L_f g_{i^*}(z, \bar{d}(\bar{t})) = 0$$

$$\left(\text{resp.} \quad \min_{d\in D}\max_{i\in \mathbb{I}(z)} L_f g_i(z, d) = \max_{i\in \mathbb{I}(z)} L_f g_i(z, \bar{d}(\bar{t})) = L_f g_{i^*}(z, \bar{d}(\bar{t})) = 0\right).$$

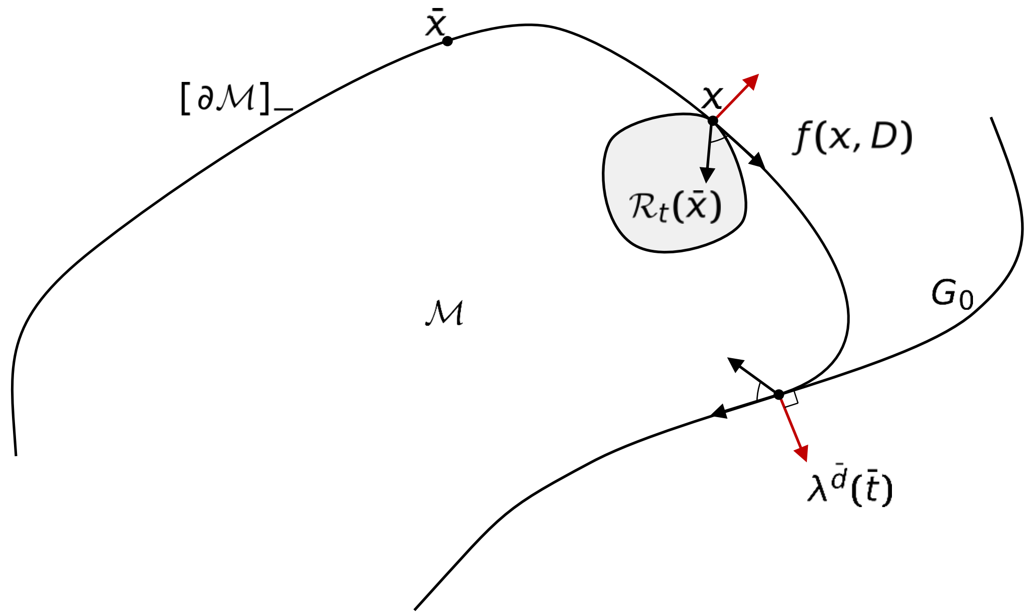
Theorem (Maximum Principle)
Let $\bar{d} \in \mathcal{D}$ be a disturbance realization such that $x^{(\bar{d}, x_0, t_0)}(t_1) \in \partial R_{t_1}(x_0)$for some $t_1 > t_0$. Then, there exists a non-zero absolutely continuous maximal solution to the adjoint such that

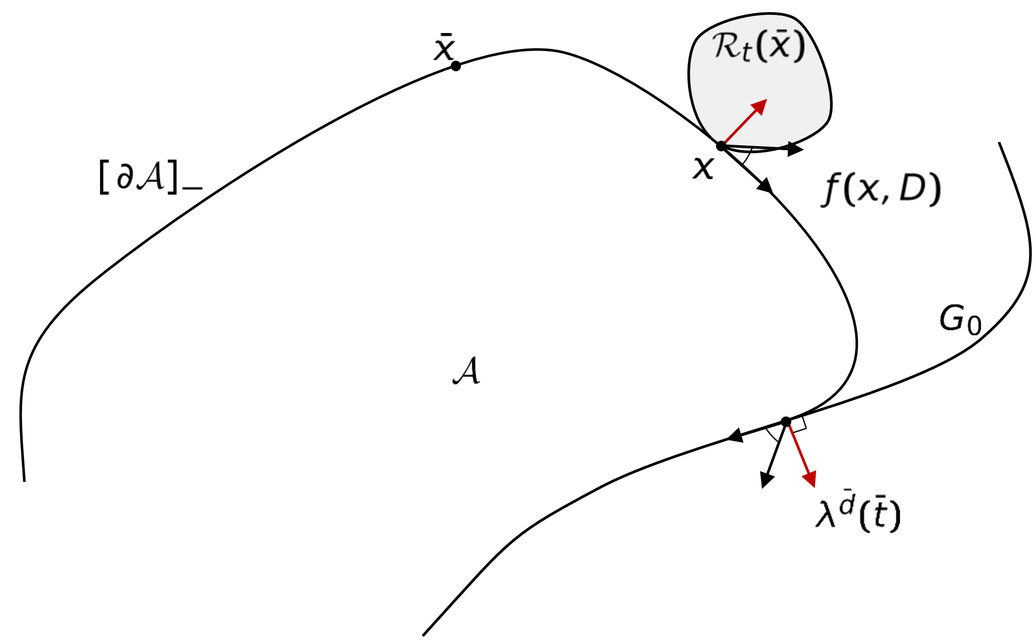
$$\max_{d\in D}\{\lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), d)\} = \lambda^{\bar{d}}(t)^T f(x^{(\bar{d})}(t), \bar{d}(t)) = \text{constant}$$

for almost every $t \in [t_0, t_1]$.

[Lee and Markus, Foundations of Optimal Control Theory, 1967]

Some interpretation

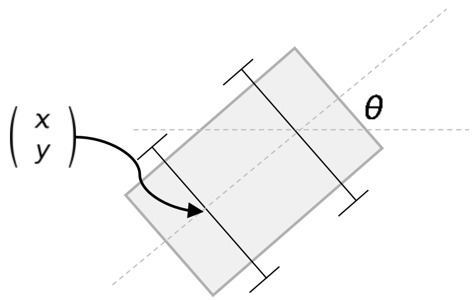


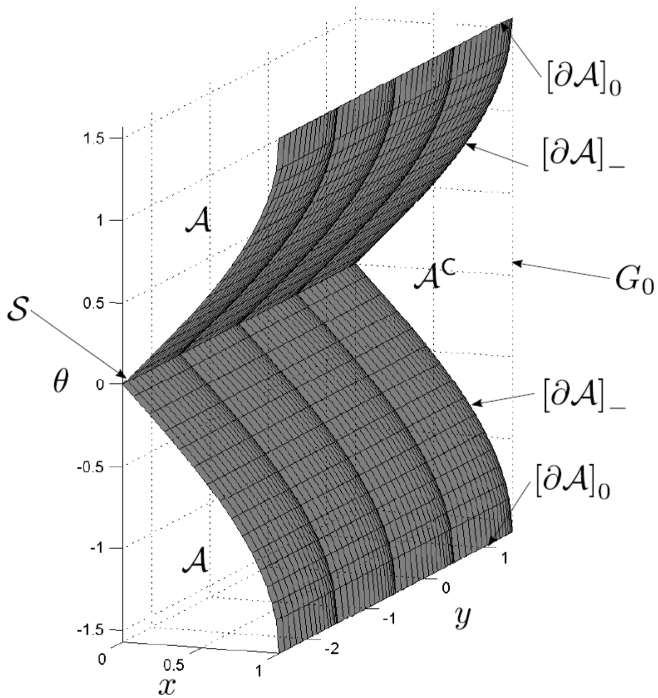


$$\mathcal{R}_t(\bar{x}) \triangleq \{x^{(d,\bar{x})}(t) : d \in \mathcal{D}\}$$

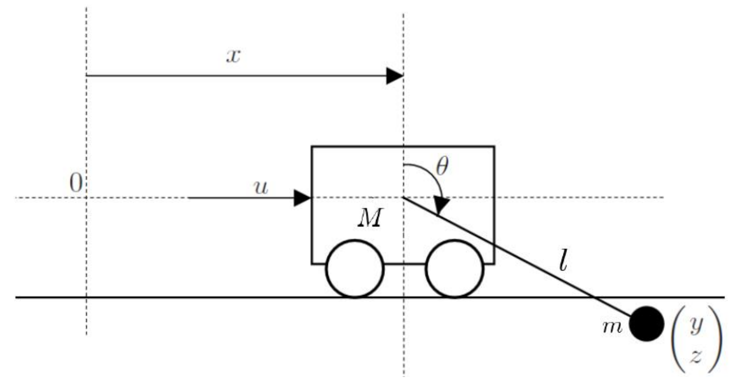
Some examples: Admissible sets

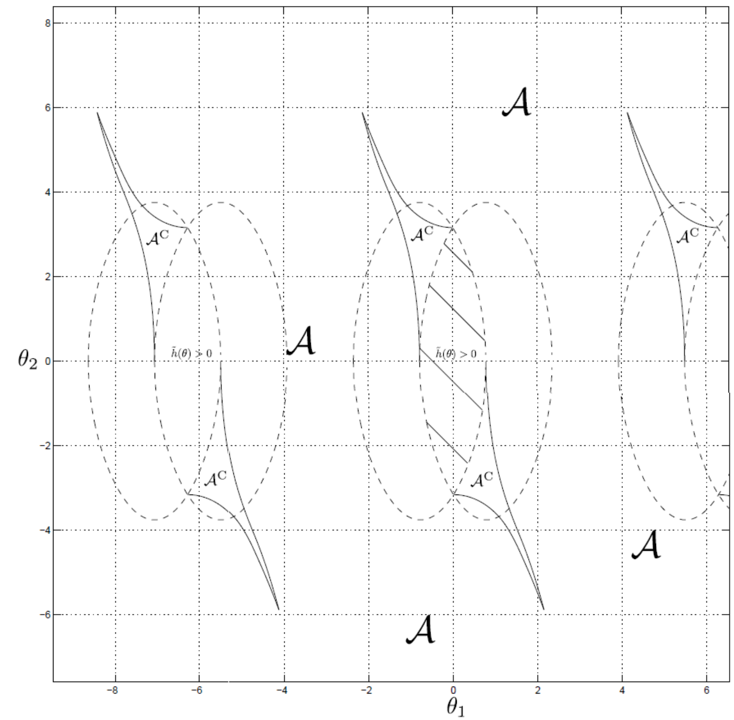
$$\dot{x} = \cos(\theta),$$
$$\dot{y} = \sin(\theta),$$
$$\dot{\theta} = u,$$
$$x \leq 1,$$
$$|u| \leq 1.$$





[Esterhuizen and Lévine: A preliminary study of barrier stopping points in constrained nonlinear systems, in Proceedings of the 19th IFAC World Congress, 2014]

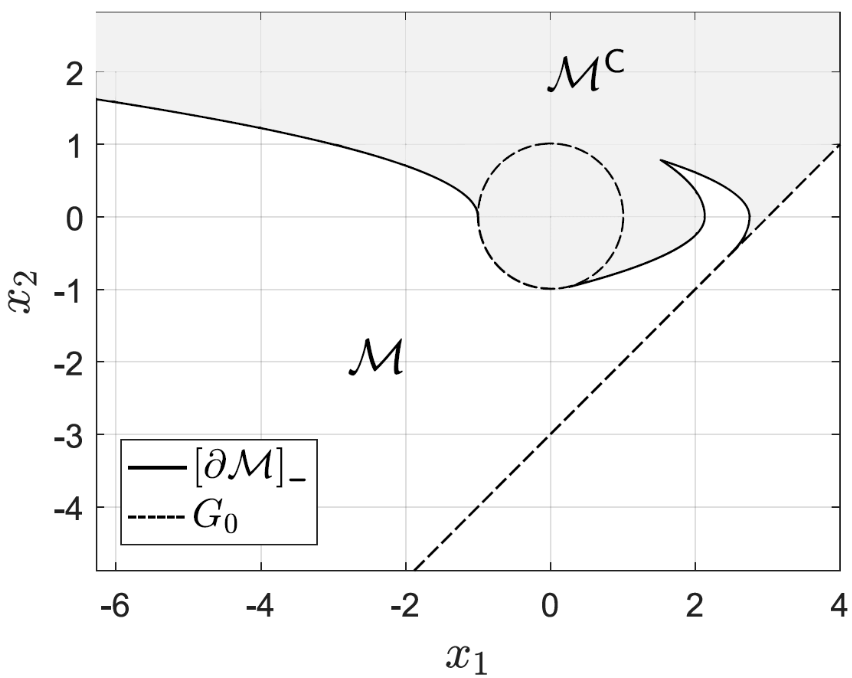


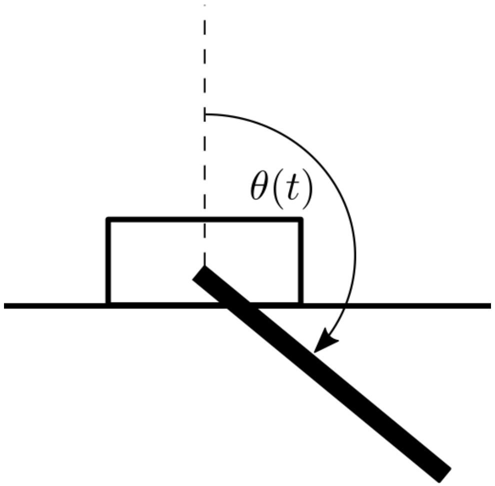


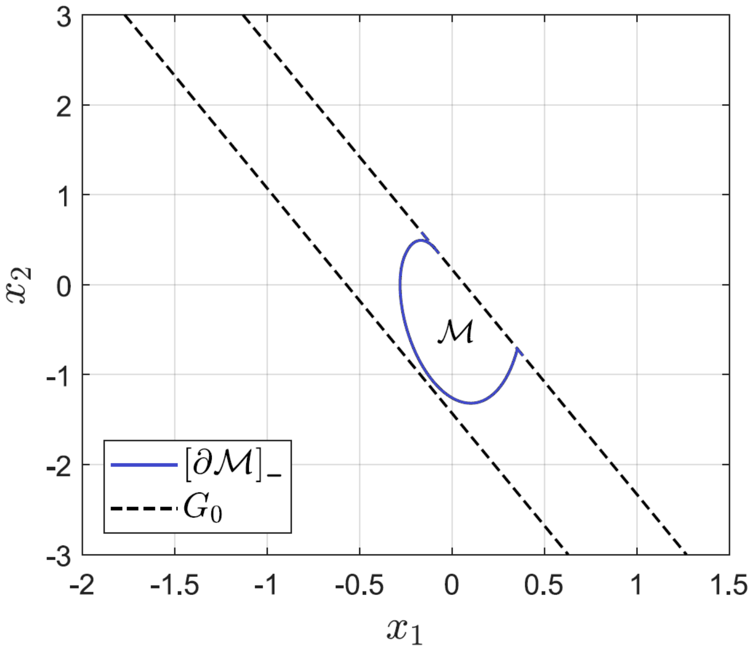
[Esterhuizen and Lévine: Barriers and potentially safe sets in hybrid systems: Pendulum with non-rigid cable, Automatica, 2016]

Some examples: MRPIs

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = d$$
$$d \in [-0.5, -0.25]$$
$$-x_1^2 - x_2^2 + 1 \leq 0$$
$$x_1 - x_2 - 3 \leq 0$$





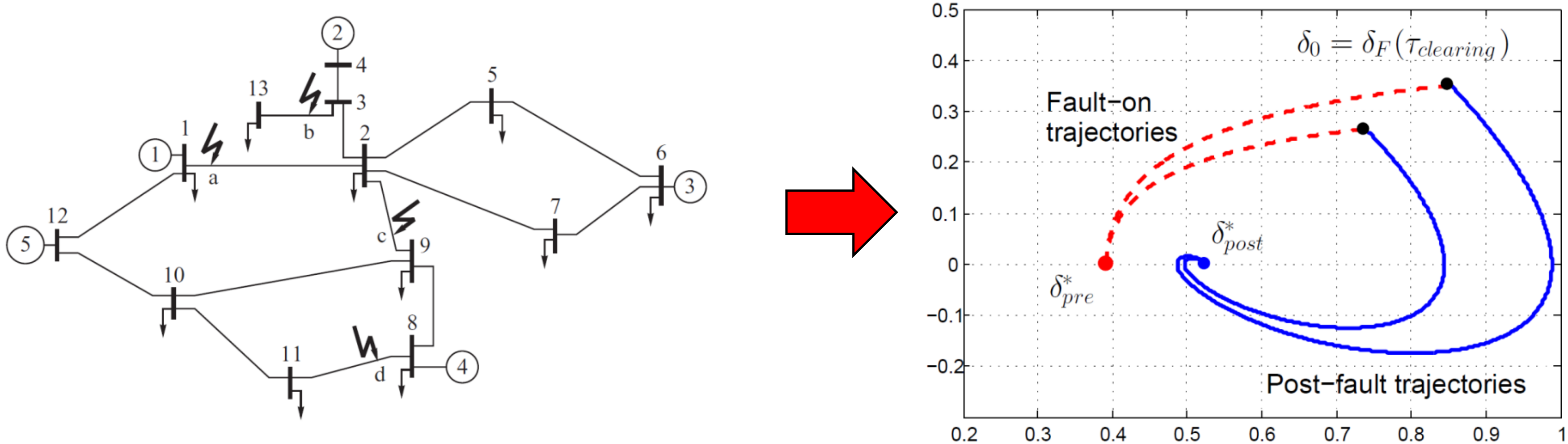


[Esterhuizen, Aschenbruck, Streif: On Maximal Robust Positively Invariant Sets in Constrained Nonlinear Systems, under review]

The Transient Stability Problem

Problem Setting

- Grid is operating in a “stable” way: rotor velocities & phase angles at all busses within acceptable bounds.
- A contingency occurs.
- Will the rotor velocities and phase angles remain in their bounds?
- Will the system evolve to a “stable” operating point?



Assumptions

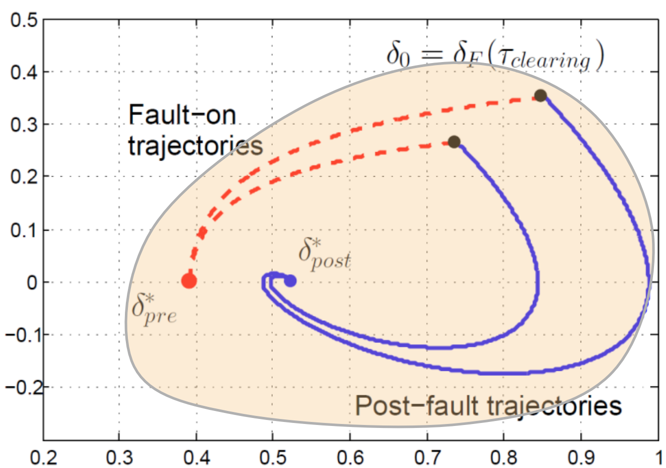
- Mechanical input torque to synchronous machines is constant.
- Power demand at loads is constant.

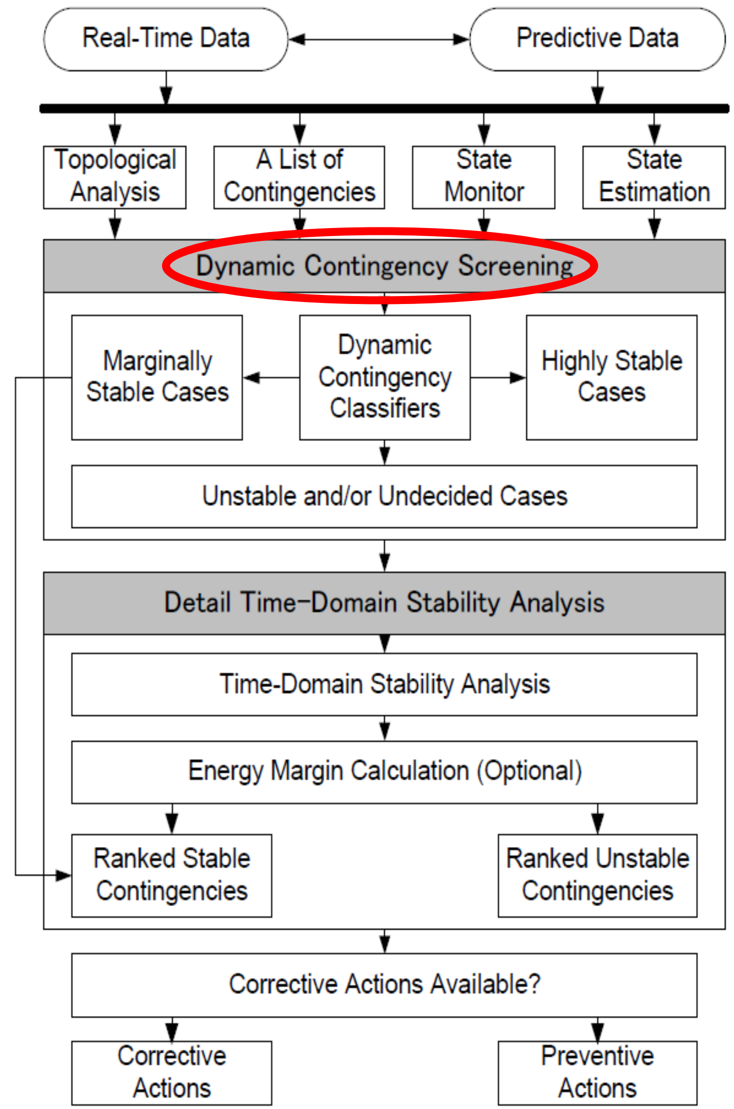
Typical system:

1. Have model & list of contingencies (10 000 +)
2. Screen: find those most likely to cause instability (≈ 1000)
3. Simulate all of them
4. Find the most critical ones
5. Do this periodically (roughly every hour)

Our contribution:

- We propose a new “set-based” approach to Dynamic Contingency Screening (DCS)

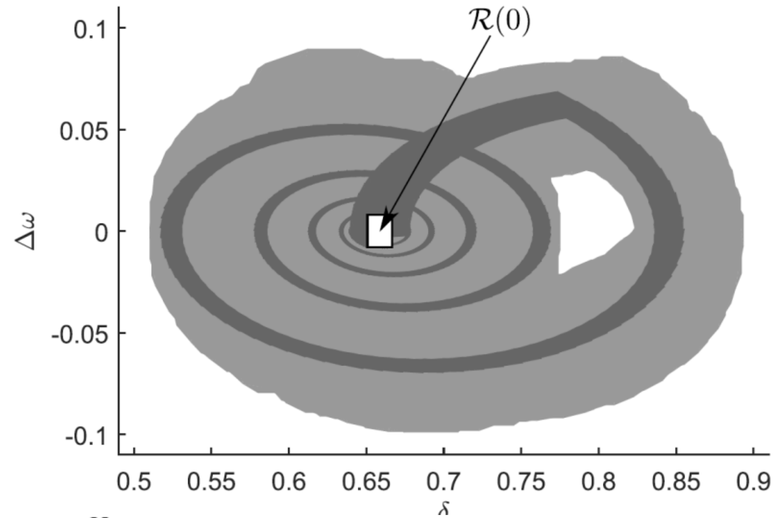


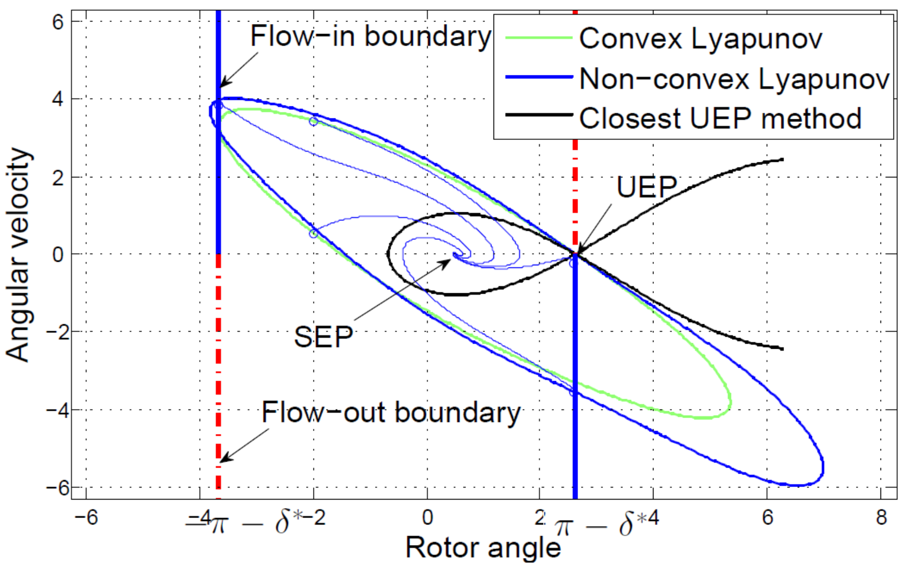


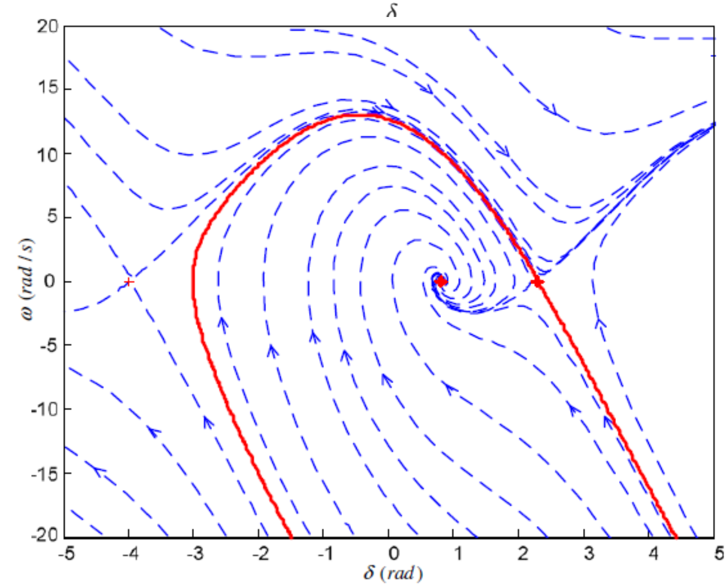
[Chiang, Tong, Tada: On-line transient stability screening of 14,000-bus models using TEPCO-BCU:Evaluations and methods, IEEE PES General Meeting, 2010]

Current DCS Approaches

- Simulation for 2 or 3 seconds
- Direct methods
  - Various Lyapunov function approaches
  - Energy functions ((UEP) approaches)
- Set-based methods (newer ideas)
  - Forward reachable sets
  - Backward reachable sets
  - Novel method: Admissible sets & MRPIs







[Turitsyn et al.: Lyapunov Functions Family Approach to Transient Stability Assessment, IEEE Transactions on Power Systems, 2016]

[Althoff et al.: Compositional transient stability analysis of power systems via the computation of reachable sets, ACC, 2017]

[Jin et al.: Reachability analysis based transient stability design in power systems, International Journal of Electrical Power & Energy Systems, 2010]

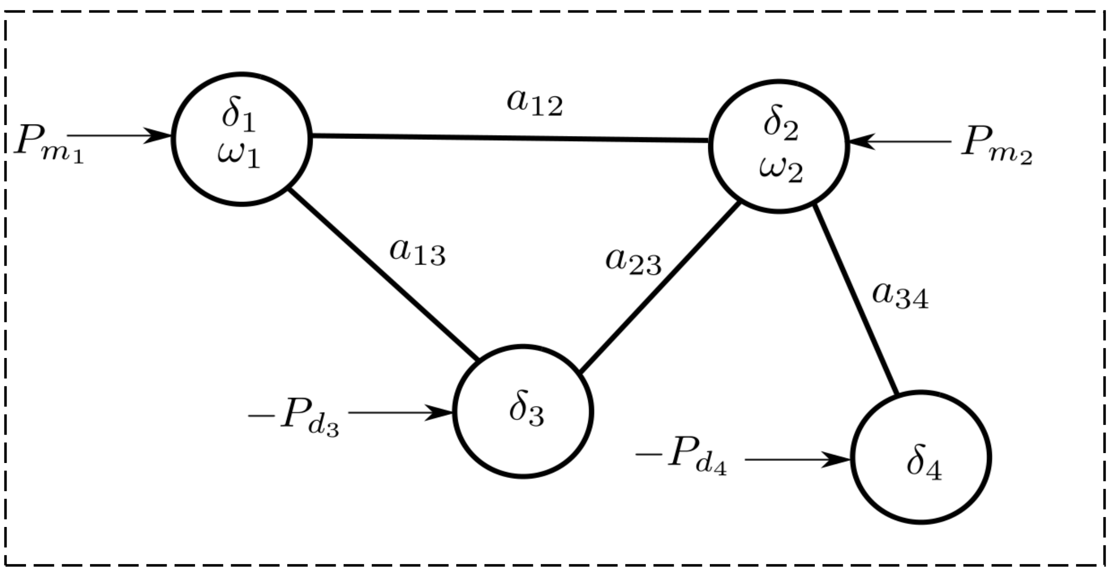
Modelling the power grid

Structure-preserving model

- An undirected graph
- Nodes represent generator/load buses
- Edges represent transmission lines
- Lines are lossless (i.e. zero conductance)
- Gen's modelled by the "swing equation"
- Loads are modelled by a first-order ODE.

$$m_k \ddot{\delta}_k + d_k \dot{\delta}_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = P_{m_k}, \quad k \in \mathcal{G}$$

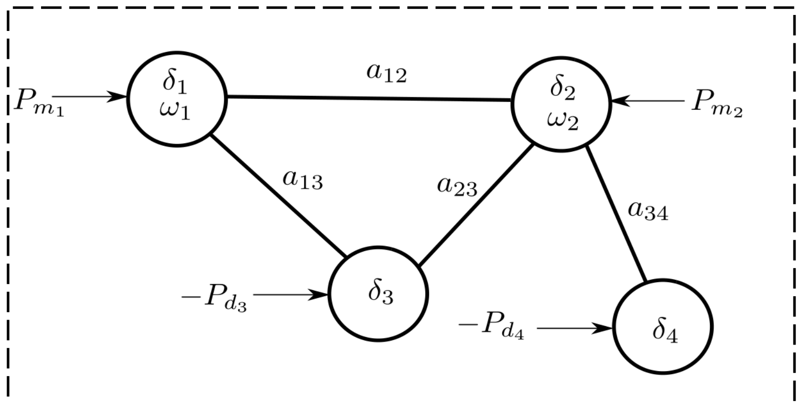
$$d_k \dot{\delta}_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = -P_{d_k}, \quad k \in \mathcal{L}$$



Analysis with the sets

Idea

- Consider each generator individually
- Impose state constraints
- These form "safe sets"
- Treat its neighbours' δ's as inputs
- Find the node's $\mathcal{M}$ and $\mathcal{A}$



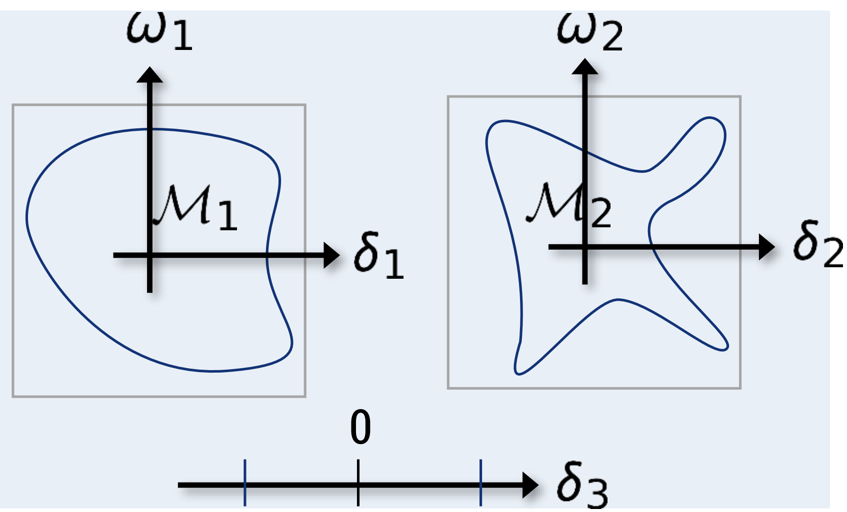
Machine 1

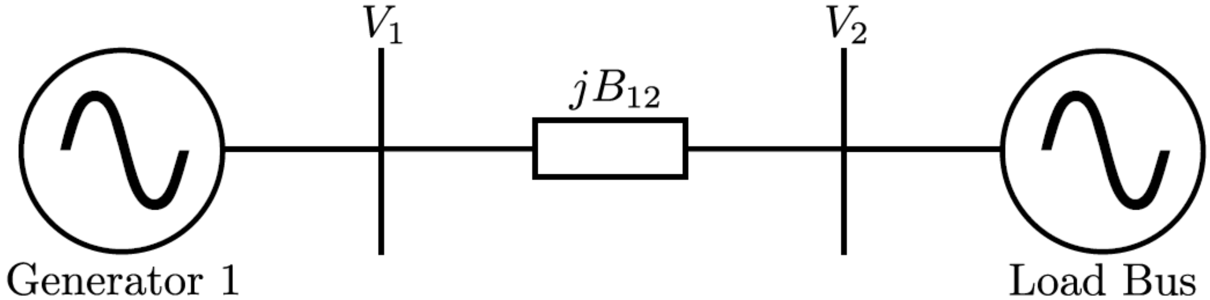
$$m_1\ddot{\delta}_1 + d_1\dot{\delta}_1 + \sum_{j\in\{2,3\}} a_{1j}\sin(\delta_1 - \delta_j) = P_{m_1}$$

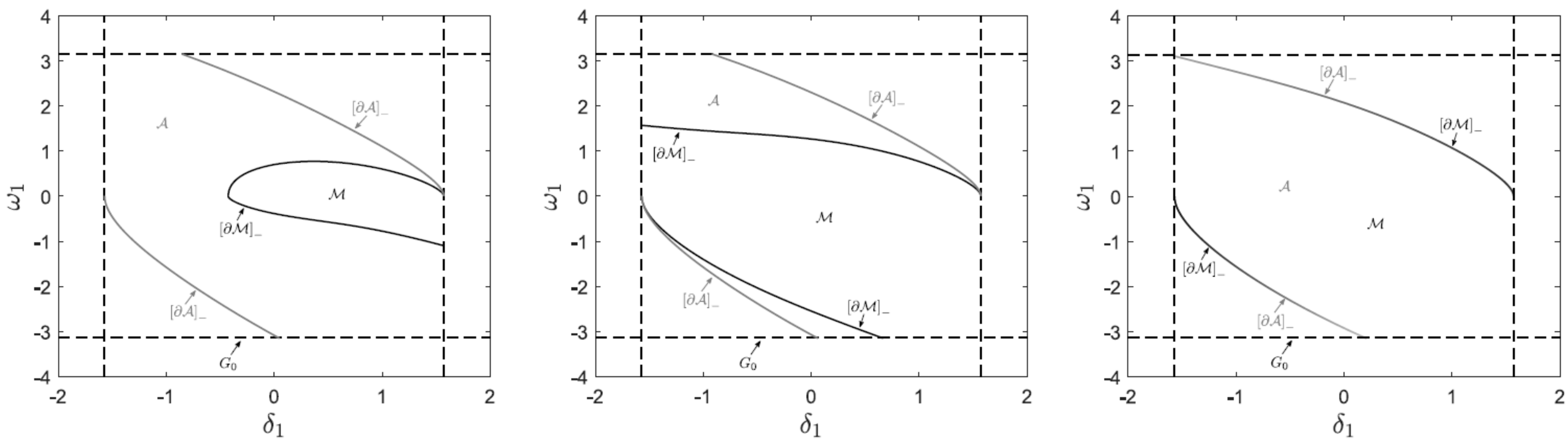
$$|\delta_1| \leq \pi/2, \quad |\omega_1| \leq \pi$$

Machine 1's neighbours

$$|\delta_2| \leq \pi/2, \quad |\delta_3| \leq \pi/2$$



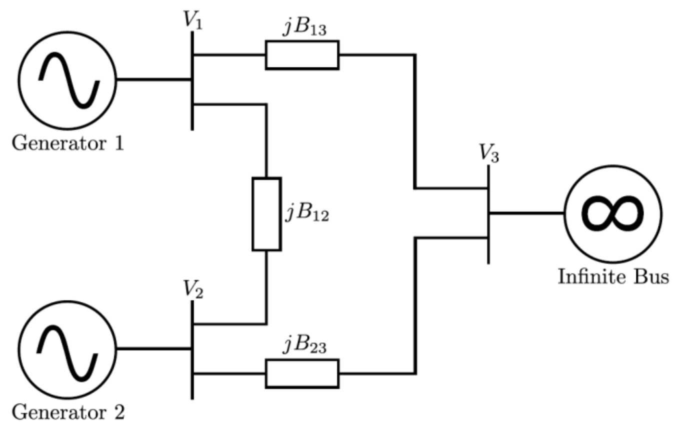


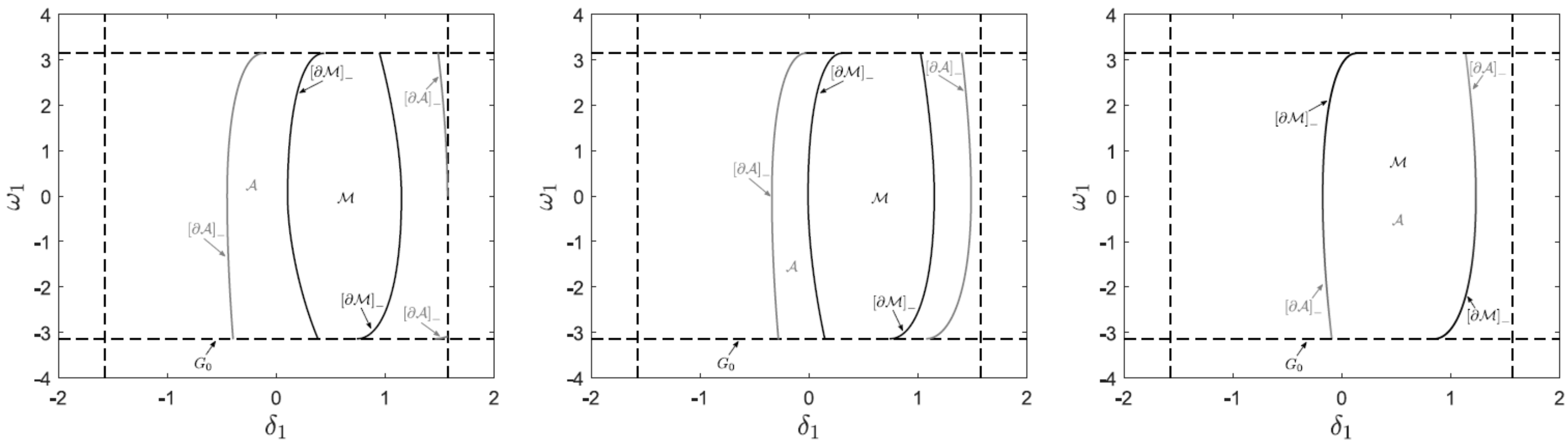


(a) $\overline{\delta}_2 = \frac{\pi}{3.7}$ (b) $\overline{\delta}_2 = \frac{\pi}{4.5}$ (c) $\overline{\delta}_2 = 0$

Fig. 3. The admissible sets and MRPIs for the two bus system with different disturbance bounds $\overline{\delta}_2$.

[Aschenbruck, Esterhuizen, Streif: Transient stability analysis of power grids with admissible and maximal robust positively invariant sets, not yet submitted]



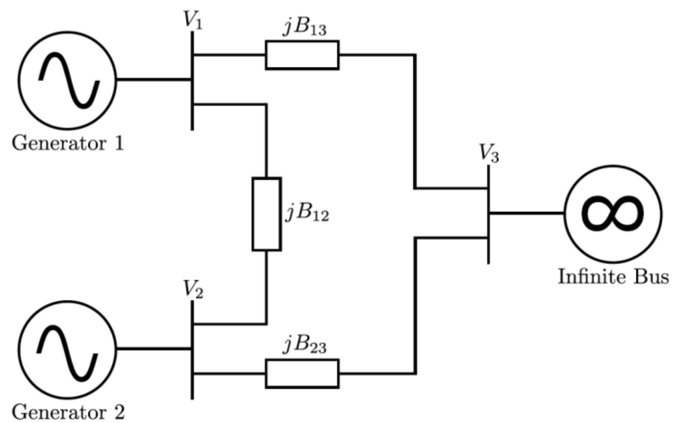


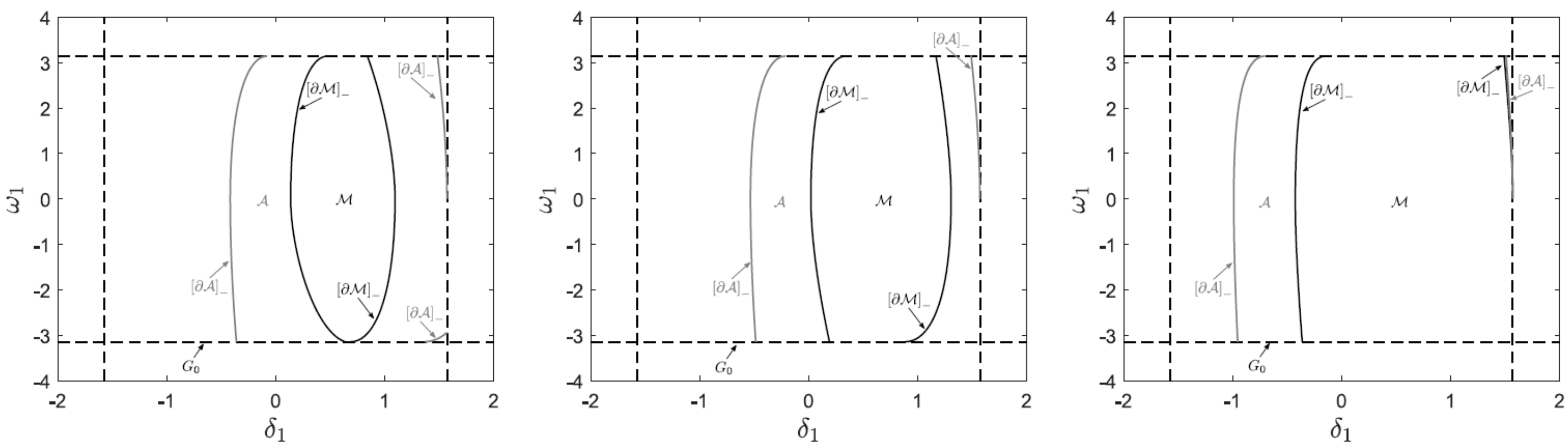
(a) $\overline{\delta}_2 = \frac{\pi}{2}$ (b) $\overline{\delta}_2 = \frac{\pi}{4}$ (c) $\overline{\delta}_2 = 0$

Fig. 5. The admissible sets and MRPIs for machine one of the double machine infinite bus system with different disturbance bounds $\overline{\delta}_2$.

[Aschenbruck, Esterhuizen, Streif: Transient stability analysis of power grids with admissible and maximal robust positively invariant sets, not yet submitted]

Examples: double machine, infinite bus



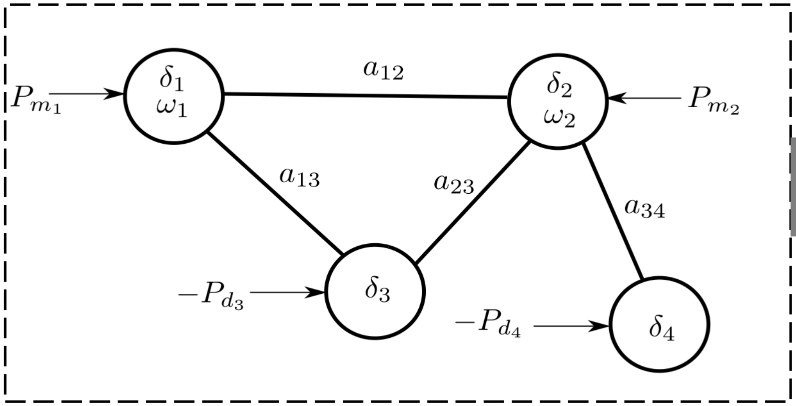


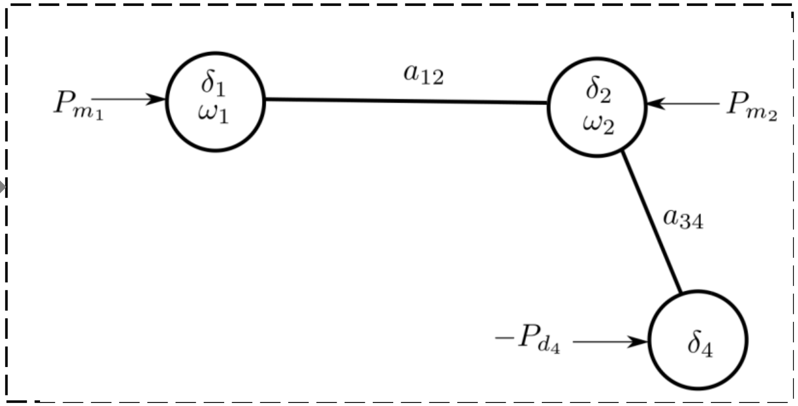
(a) $k_1 = 0.36$ (b) $k_1 = 0.5$ (c) $k_1 = 1$

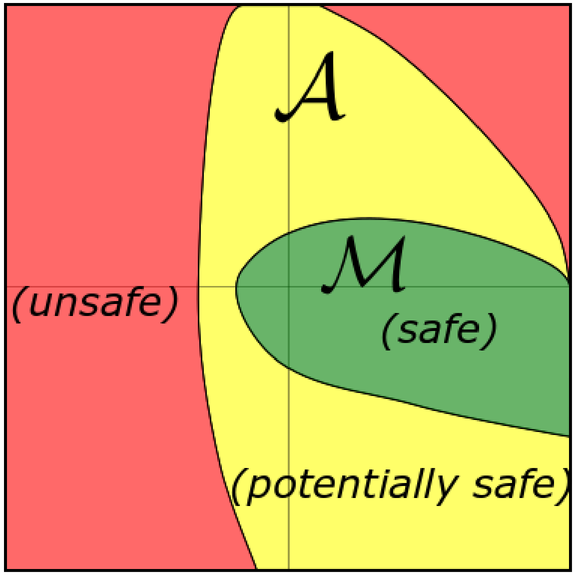
Fig. 7. The admissible sets and MRPIs for machine two of the double machine infinite bus system with different damping coefficients, k_1.

[Aschenbruck, Esterhuizen, Streif: Transient stability analysis of power grids with admissible and maximal robust positively invariant sets, not yet submitted]

To explore: reduced-order models







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